Dr. A. Betten Spring 2008

MATH 369 Linear Algebra

Assignment # 6

Problem # 32
Find a basis for the solution set of the system

$$\begin{array}{rrrcl} x_1 & -4x_2 & +3x_3 - 4_x & = & 0 \\ 2x_1 & -8x_2 & +6x_3 - 2x_4 & = & 0 \end{array}$$

Problem # 33
Find a basis and determine the dimension for each of the following: a) 2×2 matrices, b) symmetric 2×2 matrices, c) symmetric 3×3 matrices, d) $\mathbb{C}$ as vector space over $\mathbb{R}$.

Problem # 34
Find a basis for each of the following subspaces of P_4, the space of cubic polynomials:
a) $\{p(x) \in P_4 \mid p(7) = 0\}$
b) $\{p(x) \in P_4 \mid p(7) = p(5) = 0\}$
c) $\{p(x) \in P_4 \mid p(7) = p(5) = p(3) = 0\}$
d) $\{p(x) \in P_4 \mid p(7) = p(5) = p(3) = p(1) = 0\}$

Problem # 35
Find a vector $\vec{v}$ that completes the basis for the given space:
a) $\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{v}\}$ for $\mathbb{R}^2$. b) $\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{v}\}$ for $\mathbb{R}^3$. c) $\{x, 1 + x^2, \vec{v}\}$ for P_3 the set of quadratic polynomials.

Problem # 36
Verify that the set $\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid x + y + z = 1\}$ is a vector space with respect to the following structure:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 - 1 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix} \qquad r \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} rx + r - 1 \\ ry \\ rz \end{pmatrix}.$$

Find a basis.

Problem # 37
Find a basis for the row-space of the following matrix: $\begin{pmatrix} 2 & 0 & 3 & 4 \\ 0 & 1 & 1 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 0 & -4 & -1 \end{pmatrix}$

Problem # 38
Find the rank of each matrix:
a) $\begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 1 & 0 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -2 & 2 & -4 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 3 & 2 \\ 5 & 1 & 1 \\ 6 & 4 & 3 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$