Dr. A. Betten Spring 2008

MATH 369 Linear Algebra

Assignment # 10

Problem # 59

Decide whether each map is an isomorphism (if yes, prove it, if no, state a condition that it fails to satisfy). We denote by $\mathcal{M}_{m\times n}$ the set of $m \times n$ real matrices.

a) $f : \mathcal{M}_{2\times 2} \to \mathbb{R}$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto ad - bc$$

b)$f : \mathcal{M}_{2\times 2} \to \mathbb{R}^4$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} a+b+c+d \\ a+b+c \\ a+b \\ a \end{pmatrix}$$

c)$f : \mathcal{M}_{2\times 2} \to P^4$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto c + (d+c)x + (b+a)x^2 + ax^3$$

d)$f : \mathcal{M}_{2\times 2} \to P^4$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto c + (d+c)x + (b+a+1)x^2 + ax^3$$

Problem # 60

Consider the vector space $V = \mathbb{R}^n$ with standard basis $\mathbf{e}_1, \dots, \mathbf{e}_n$. Let $\mathbf{e}_i^*$ be the linear map from V to $\mathbb{R}$ whose image on the basis is

$$\mathbf{e}_i^*(\mathbf{e}_j) = \delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $\mathbf{e}_1^*, \dots, \mathbf{e}_n^*$ is a basis of a vector space V^* of dimension n.

b) Let $\mathrm{Hom}(V, \mathbb{R})$ be the vector space of all linear functions from V to $\mathbb{R}$ (called the dual space). Show that $\mathrm{Hom}(V, \mathbb{R}) = V^*$.

c) Let $\mathbf{b}_1 = (1, 1, 1), \mathbf{b}_2 = (-1, 0, 1), \mathbf{b}_3 = (0, 1, -1)$ be a basis of $\mathbb{R}^3$. Compute a basis $\mathbf{b}_1^*, \mathbf{b}_2^*, \mathbf{b}_3^*$ for the dual space $\mathrm{Hom}(V, \mathbb{R})$ satisfying $\mathbf{b}_i^*(\mathbf{b}_j) = \delta_{i,j}$ (such a basis is called a dual basis). Your goal is to express the $\mathbf{b}_i^*$ in terms of the $\mathbf{e}_j^*$.