Practice

Problem # 1
(a) Find integers $s$ and $t$ such that $s123 + t203 = 1$.
(b) Find $19^{-1} \mod 97$.

Problem # 2
Solve the following system of congruences:

\[
\begin{align*}
  x &\equiv 1 \mod 2 \\
  x &\equiv -1 \mod 19 \\
  x &\equiv 14 \mod 17
\end{align*}
\]

Problem # 3
Suppose you work mod 25 instead of mod 26 for affine ciphers. How many keys are possible?

Problem # 4
Compute

a) $\Phi(24683)$

b) $\Phi(111222)$

Problem # 5
a) Compute the multiplication table (a.k.a. TIMES$_{12}$) for $U_{12}$. Is it cyclic? Prove!
b) Compute the multiplication table (a.k.a. TIMES$_{10}$) for $U_{10}$. Is it cyclic? Prove!

Problem # 6
Compute the last three digits of $7^{803}$
Problem # 7
The following ciphertext was encrypted by an affine cipher mod 26:

\[ fzmnah. \]

The plaintext starts \( g \) and ends in \( y \). Decrypt the message.

Problem # 8
Compute the inverse of the function \( f(x) = 22x + 18 \mod 71 \).

Problem # 9
Encipher “Influenza Cases are increasing” using the Playfair Cipher with Keyword “President Frank”.

Problem # 10
a) Turn 666 into binary.
b) Turn \((101011)_2\) into decimal.
c) Turn 777 into base-3.
d) Turn \((10101101)_2\) into base-16 (using \( a = 10, b = 11, c = 12, d = 13, e = 14, f = 15 \)).