Problem # 5
Compute \( \gcd(4883, 4369) \) and express it in the form \( s \cdot 4883 + t \cdot 4369 \)

Problem # 6
a) Express 9776 in binary.
   b) Express \((10010101)_2\) in decimal.
   c) Express \((5416)_7\) in base 11

Problem # 7
You intercept the ciphertext “OFJDFOHFXOL”, which was enciphered using an affine transformation of single-letter plaintext units in the 27-letter alphabet (with blank=26). You know that the first word is “T” (“T” followed by blank). Determine the enciphering key, and read the message.

Problem # 8
Divide \(2^{9439}\) by 107. What is the remainder?

Problem # 9
Find the last 2 digits of \(123^{62}\). *Hint:* You may use the Chinese Remainder Theorem.

Problem # 10
(a) Solve \(7d \equiv 1 \mod 30\)
   (b) Suppose you write a message as a number \(m \mod 31\). Encrypt \(m\) as \(m^7 \mod 31\). How would you decrypt? *Hint:* Decryption is done by raising the ciphertext to a power \(\mod 31\). Fermat’s theorem will be useful.

Problem # 11
Let \(p\) be a prime and \(a\) an integer not divisible by \(p\). Explain in your own words why the sets
\[
\{1, 2, 3, \ldots, p-1\}
\]
and
\[
\{1a, 2a, 3a, \ldots, (p-1)a\}
\]
are the same (when considered \(\mod p\)).