Dr. A. Betten Spring 2006

M502 Combinatorics II

exercise sheet # 1

Exercise # 1 (5 points)
The *Clebsch graph* is defined as follows: vertices are the subsets of a five set, say $\{a, b, c, d, e\}$ of *even* size. Two subsets are *adjacent* if and only if they differ in four elements. That is, their *summetric difference* has cardinality 4. Here, the symmetric difference of two sets X and Y, denoted as $X \Delta Y$, is defined as $X \Delta Y := (X \setminus Y) \cup (Y \setminus X)$, e.g. $\{a, b\} \sim \{a, c, d, e\}$, since we have $\{a, b\}\Delta\{a, c, d, e\} = \{b, c, d, e\}$, a four element set. Verify that the Clebsch graph is a strongly regular graph $s.r.g.(16, 5, 0, 2)$ (try not to forget cases!).

Exercise # 2 (5 points)
Find automorphisms of the Petersen graph. You only need to give generators. The larger the order of the generated group the better. Hint: the group order is 120. Use GAP to verify the group order. Go to Alexander Hulpke's Web page for more info on GAP. This is an example for using GAP.

```
gap>
gap> g := [(1,2),(1,2,3,4,5)];
[ (1,2), (1,2,3,4,5) ]
gap> G := Group(g);
Group([ (1,2), (1,2,3,4,5) ])
gap> Size(G);
120
gap>
```

We create a vector (list) "g" of generators, namely the two permutations

$$(1, 2) \text{ and } (1, 2, 3, 4, 5)$$

and then we create the group "G" which is generated by the elements in "g". The last command "Size" gives the order of the group, which is 120 in this case.

due to Monday, 2/6/06.