Dr. A. Betten Spring 2007

M460 Information and Coding Theory

homework sheet # 2

Problem # 1
Show that the entropy function satisfies

$$H(p_1, \ldots, p_{i-1}, qp_i, (1-q)p_i, p_{i+1}, \ldots, p_n) = H(p_1, \ldots, p_n) + p_i H(q, 1-q).$$

for a probability distribution $p_1, \ldots, p_n$ and $0 \le q \le 1$.

Problem # 2
Using the estimates of frequencies of letters given in M360, give a rough and tough estimate for the entropy of English language text. (Do not split hairs, use the frequency groups).

Problem # 3
Let $\mathcal{S}$ be a source with three symbols with probabilities $p_1 \ge p_2 \ge p_3$. Show that the expected length of the binary Huffman code for $\mathcal{S}$ is $2 - p_1$.

Problem # 4
Suppose we toss a fair coin, and if the outcome is heads, we toss it again. How much uncertainty is there in the outcome?

Problem # 5
Suppose we toss a fair coin and roll a fair die. Do we get more information from this experiment or from the experiment of tossing three fair coins? four fair coins?

Problem # 6
How much information do we get by sampling from a deck of cards if
a) each card is equally likely to be drawn?
b) the black cards are twice as likely to be drawn as the red cards?

Problem # 7
Suppose that we roll a fair die that has two faces numbered 1, two faces numbered 2, and two faces numbered 3. Then we toss a fair coin the number of times indicated by the number on the die. How much information do we get by this procedure?

Problem # 8
Let $\mathcal{S} = \{0, 1, 2\}$ be a source, with $P(0) = p, P(1) = q, P(2) = 1-p-q$. Repeatedly performing the experiment of sampling this source until a 2 appears produces another source with alphabet $\{a_1 \cdots a_k 2 \mid a_i \in \mathcal{S}, a_i \neq 2\}$. Calculate the probability distribution and the entropy of this source.

Problem # 9
Find a Huffman encoding of the given probability distribution P for the given radix r when $P = (0.3, 0.05, 0.03, 0.02, 0.3, 0.1, 0.15, 0.05)$.
a) $r = 2$
b) $r = 3$
c) $r = 4$
d) $r = 5$

Problem # 10
The inhabitants of an island tell the truth one third of the time. They lie with probability 2/3. On an occasion, after one of them made a statement, you ask another "was that statement true"? and he says "yes". What is the probability that the statement was indeed true?

Problem # 11
There are eleven urns labeled by $u \in \{0, 1, \ldots, 10\}$, each containing ten balls. Urn u contains u black balls and $10 - u$ white balls. Fred selects an urn u at random and draws N times with replacement from that urn, obtaining n_B blacks and $N - n_B$ whites. Fred's friend, Bill, looks on. If after $N = 10$ draws $n_B = 3$ blacks have been drawn, what is the probability that the urn Fred is using is urn u, from Bill's point of view? (Bill does not know the value of u.)

Problem # 12
On a gameshow, a contestant is told the rules as follows: There are three doors, labeled 1,2,3. A single prize has been hidden behind one of them. You get to select one door. Initially your chosen door will *not* be opened. Instead, the gameshow host will open one of the other two doors, and he *will do so in such a way as not to reveal the prize.* For example, if you choose door 1, he will then open one of doors 2 and 3, and it is guaranteed that he will choose which one to open so that the prize will not be revealed. At this point, you will be given a fresh choice of door: you can either stick with your first choice, or you can switch to the other closed door. All the doors will then be opened and you will receive whatever is behind your final choice of door. Imagine that the contestant chooses door 1 first. Then the gameshow host opens door 3, revealing nothing behind the door, as promised. Should the contestant

a) stick with door 1, or

b) switch to door 2, or

c) does it make no difference?

due Monday, Feb 12. Attn: Midterm 1: 2/14/07.