

M460 Information and Coding Theory

homework sheet # 1

Problem # 1

For the following binary codes, determine:

- Whether the code is *uniquely decodable*. If not, exhibit two source messages with the same code.
- Whether the code is *instantaneous*. If not, can you design an instantaneous code with the same lengths of code words?

	code A	code B	code C	code D	code E	code F	code G	code H
s_1	000	0	0	0	0	0	01	1010
s_2	001	01	10	10	10	100	011	001
s_3	010	011	110	110	1100	101	10	101
s_4	011	0111	1110	1110	1101	110	1000	0001
s_5	100	01111	11110	1011	1110	111	1100	1101
s_6	101	011111	111110	1101	1111	001	0111	1011

Problem # 2

For the following code word lengths:

Code	Lengths
A	2 2 2 4 4 4
B	1 1 2 3 3
C	1 1 2 2 2 2

- Can an instantaneous binary code be formed? If so, give an example of such a code.
- Can an instantaneous ternary code be formed? If so, give an example of such a code.
- If neither a binary nor a ternary code can be formed find the smallest number of code symbols that will allow a code to be formed. Give an example of such a code.

Problem # 3

Compute the binary entropy of a source with propabilities

- $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}$,
- $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}, \frac{1}{24}$.

Problem # 4

Compute the binary entropy of the random variable which counts the number of heads in flipping three fair coins.

Problem # 5

Compute the binary entropy of the random variable which counts the sum of three dice.

Problem # 6

What is the maximum number of words in an *instantaneous* binary code with maximum word length 5?

Problem # 7

A 6-symbol source has the following statistics and suggested *binary* and *ternary* codes:

s_i	p_i	Code A	Code B
s_1	0.3	0	00
s_2	0.2	10	01
s_3	0.1	1110	02
s_4	0.1	1111	10
s_5	0.2	1100	11
s_6	0.1	1101	12

- What is the *efficiency* of binary code A? Recall that the efficiency of a code is the entropy over the expected length.
- What is the *efficiency* of ternary code B?
- Can you design a more efficient binary code and, if so, what is the efficiency of your code?
- Can you design a more efficient ternary code and, if so, what is the efficiency of your code?
- Which is the most efficient code: binary or ternary?

Problem # 8

You are given 12 balls, all equal in weight except for one that is either heavier or lighter. You are given a two-pan balance to use. In each use of the balance you may put any number of the 12 balls on the left pan, and the same number on the right pan, and push a button to initiate the weighing. There are three possible outcomes: either the weights are equal, or the balls on the left are heavier, or the balls on the left are lighter. Your task is to design a strategy to determine which is the odd ball and whether it is heavier or lighter than the others in as few uses of the balance as possible. Draw your solution in the form of a tree.

due Monday, Jan 29.