

M460 Information and Coding Theory

exercise sheet # 1

Exercise # 1

(3 points)

An urn contains K balls, of which B are black and $W = K - B$ are white. Fred draws a ball at random from the urn and replaces it, N times.

- a) What is the probability distribution of the number of times a black ball is drawn, n_B ?
- b) What is the expectation of n_B ? What is the variance of n_B ? What is the standard deviation of n_B ? Give numerical answers for the case $N = 5$ and $N = 400$, when $B = 2$ and $K = 10$.

Exercise # 2

(3 points)

There are eleven urns labeled by $u \in \{0, 1, \dots, 10\}$, each containing ten balls. Urn u contains u black balls and $10 - u$ white balls. Fred selects an urn u at random and draws N times with replacement from that urn, obtaining n_B blacks and $N - n_B$ whites. Fred's friend, Bill, looks on. If after $N = 10$ draws $n_B = 3$ blacks have been drawn, what is the probability that the urn Fred is using is urn u , from Bill's point of view? (Bill does not know the value of u .)

Exercise # 3

(4 points)

On a gameshow, a contestant is told the rules as follows: There are three doors, labeled 1,2,3. A single prize has been hidden behind one of them. You get to select one door. Initially your chosen door will *not* be opened. Instead, the gameshow host will open one of the other two doors, and he *will do so in such a way as not to reveal the prize*. For example, if you choose door 1, he will then open one of doors 2 and 3, and it is guaranteed that he will choose which one to open so that the prize will not be revealed. At this point, you will be given a fresh choice of door: you can either stick with your first choice, or you can switch to the other closed door. All the doors will then be opened and you will receive whatever is behind your final choice of door. Imagine that the contestant chooses door 1 first. Then the gameshow host opens door 3, revealing nothing behind the door, as promised. Should the contestant

- a) stick with door 1, or
- b) switch to door 2, or
- c) does it make no difference?

due Friday, 1/26/06.