Dr. A. Betten Fall 2005

M360 Mathematics of Information Security

exercise sheet # 5

Exercise # 1 (2 points)

a) Convert 966 into binary, convert $(110101011)_2$ into decimal.

b) Convert $(54265)_7$ into base 13.

Exercise # 2 (2 points)
Divide 2^{10203} by 101. What is the remainder?

Exercise # 3 (2 points)
Find the last 2 digits of 123^{562}.

Exercise # 4 (2 points)

a) Evaluate 7^7 mod 4.

b) Use part a) to find the last digit of 7^{7^7}. Note a^{b^c} means $a^{(b^c)}$ since the other possible interpretation would be $(a^b)^c = a^{bc}$, which is written more easily without a second exponentiation. Hint: the last digit is computing mod 10. Then use Chinese Remainder Theorem.

Exercise # 5 (2 points)
Let $U_n - \{i \in \mathbb{Z} \mid 1 \leq i \leq n, \ \gcd(i,n) = 1\}$. Let $\phi(n) = |U_n|$ be the Euler-function. Show that for a prime p and a positive integer a we have $\phi(p^a) = p^a - p^{a-1}$.

due to Friday, 10/14/05.