Dr. A. Betten Fall 2005

M360 Mathematics of Information Security

exercise sheet # 1

Exercise # 1 (1 points)
Let $f : \mathbb{R} \to \mathbb{R}, f(x) = \frac{3x-4}{x-1}$. Is f one-to-one? Is f onto?

Exercise # 2 (1 points)
Let $f : \mathbb{N} \to \mathbb{N}, f(n) = n + 1$,

$$g : \mathbb{N} \to \mathbb{N}, g(n) = \begin{cases} 0 & \text{if } n = 0, \\ n-1 & \text{if } n \geq 1. \end{cases}$$

Are f and g inverses of each other? Explain!

Exercise # 3 (1 points)
Determine whether $f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$, given by

$$f(a/b, c/d) = (a + c)/(b + d)$$

is a function.

Exercise # 4 (1 points)
For $f : X \to Y$, and $W \subseteq Y$, define $f^{-1}(W) = \{x \in X : f(x) \in W\}$. Let $f : \mathbb{R} \to S^1$, where $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ is the unit circle, $f(x) = e^{2\pi i x}$ (where $i = \sqrt{-1}$). What is $f^{-1}(\{1\})$?

Exercise # 5 (6=1+1+1+3 points)
We consider functions $f_i : (0, 1) \to (0, 1)$. Let

$$f_1(x) = 1/x, \quad f_2(x) = 1 - x.$$

a) What is $f_3 = f_1 \circ f_2$?

b) What is $f_4 = f_2 \circ f_1$?

c) What is $f_5 = f_1 \circ f_4$?

d) Let $f_0(x) = x$. For $i = 0, 1, \ldots, 5$ and $j = 0, 1, \ldots, 5$, compute $f_i \circ f_j$. Make a table, the (i, j)-th entry of which is k where $f_i \circ f_j = f_k$.

due to Friday, August 30.