Dr. A. Betten Fall 2006

M301 Introduction to Combinatorial Theory

homework sheet # 7

Problem # 1

FBI Special Agent Hwang is working with 5 informants who have infiltrated organized crime. She needs to make arrangements for the informants to communicate with each other, either directly or through others, but never in groups of more than two. For reasons of security, the number of meeting places must be kept as small as possible. Furthermore, each pair of informants has been assigned a danger rating (given in the table below) which indicates the risk involved in their being seen together. How can Special Agent Hwang arrange communication so as to minimize the danger? Assume the danger is proportional to the sum of the danger ratings of the individuals who meet directly.

	Jones	Brown	Hill	Ritt	Chen
Jones	-	3	4	5	2
Brown	3	-	3	1	4
Hill	4	3	-	2	3
Ritt	5	1	2	-	4
Chen	2	4	3	4	-

Problem # 2

Give an example of a connected weighted graph (not a tree) where the same edge is part of every minimal spanning tree and every maximal spanning tree.

Problem # 3

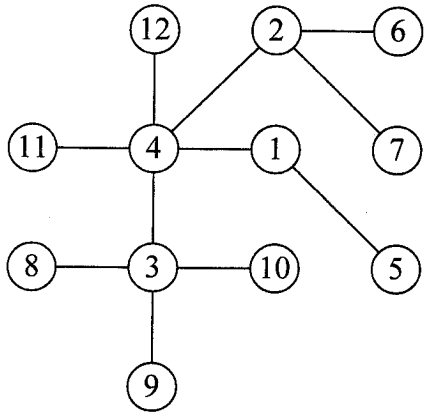
A "bad triple" in a tournament is a set of three vertices $\{a, b, c\}$ such that a beats b, b beats c, and c beats a. Show that the number of bad triples is equal to

$$\binom{n}{3} - \sum_{i=1}^{n} \binom{s_i}{2},$$

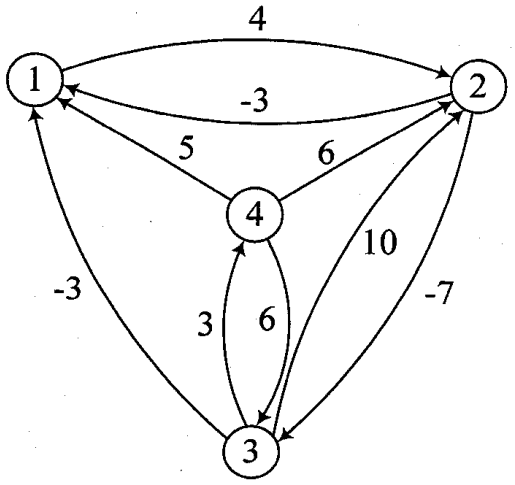
where $(s_1, \ldots, s_n)$ is the score sequence (the score s_i of vertex i is the number of out neighbors of that vertex). Deduce that the maximum number of bad triples if $\frac{1}{4}\binom{n+1}{3}$.

Problem # 4

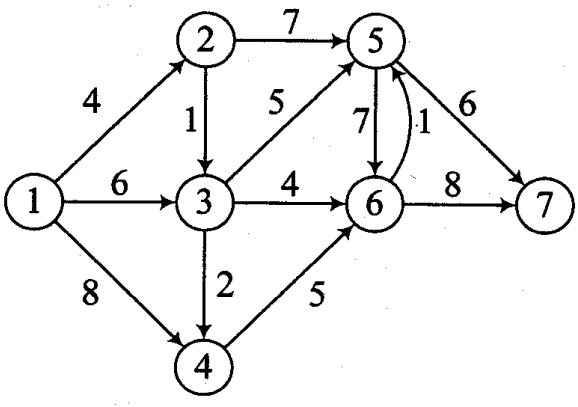
Compute the Prüfer code of the following tree:



Problem # 5
Using the algorithm of Floyd-Warshall, compute all distances in the following graph.



Problem # 6
Using Dijkstra's algorithm, compute the distances from vertex 1 in the following graph.



not collected