Dr. A. Betten Fall 2006

# M301 Introduction to Combinatorial Theory

homework sheet # 6

**Problem # 1**
Construct three nonisomorphic graphs with degree sequence $(5, 3, 2, 2, 1, 1, 1, 1)$. *Hint:* One of them is disconnected.

**Problem # 2**
Draw all possible chemical isomers having formula $C_5H_{12}$. *Hint:* A $C$-atom corresponds to a vertex of degree 4, an $H$-atom corresponds to an end vertex. Two molecules are called isomers if the corresponding graphs are isomorphic.

**Problem # 3**
Imagine that you are positioned at the origin of the $(x, y)$-plane and wish to travel to the point $(m, n)$ in the first quadrant in a sequence of special moves. Each such move consists of moving either one unit to the right or one unit up. Thus, if you are at the point $(a, b)$, where $a$ and $b$ are positive integers, you may move to either $(a + 1, b)$ or $(a, b + 1)$. It should be clear that it will take a total of $m + n$ moves to get from $(0, 0)$ to $(m, n)$. Explain why there are exactly $\binom{m+n}{m}$ ways in which to do this.

**Problem # 4**
Draw a map which is not 3-colorable.

**Problem # 5**
Construct all tournaments on 4 vertices up to isomorphism. *Hint:* there are four of them.

**Problem # 6**
Let $X = \{v_1, v_2, \ldots, v_n\}$. Define the *Hamming graph* $\mathcal{H}_X$ as follows. $V(\mathcal{H}_X) = 2^X$, i.e., vertices are subsets of $X$. Two subsets $A$ and $B$ of $X$ are adjacent in $\mathcal{H}_X$ whenever $(A \setminus B) \cup (B \setminus A)$ has size one.
(a) Show that $\mathcal{H}_X$ is isomorphic to $\mathcal{H}_Y$ whenever $|X| = |Y|$. Therefore, the Hamming graph $\mathcal{H}_X$ may simply be called $\mathcal{H}_n$ for $n = |X|$. It is also known as the Hamming graph of order $n$.
(b) Show that $\mathcal{H}_n$ is regular and compute the degree $k$ of a vertex.
(c) Given two adjacent vertices $A$ and $B$ of $\mathcal{H}_n$, how many common neighbors do $A$ and $B$ have?
(d) Given two nonadjacent vertices $A$ and $B$ of $\mathcal{H}_n$, how many common neighbors do $A$ and $B$ have?
(e) Show that $\mathcal{H}_4$ is Hamiltonian by exhibiting a Hamiltonian cycle.
(f) Show that $\mathcal{H}_n$ for $n \geq 3$ is Hamiltonian by exhibiting a Hamiltonian cycle.
(g) Compute the automorphism group of $\mathcal{H}_n$ (order, generators).
(h) What is the distance in the graph between vertices $A$ and $B$?

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