

M301 Introduction to Combinatorial Theory

homework sheet # 5

Problem # 1

Show that $1^3 + 3^3 + 5^3 + \cdots + (2n - 1)^3 = n^2(2n^2 - 1)$.

Problem # 2

We are looking for the number of ways of changing a dollar bill into coins (pennies, nickels, dimes, quarters, or a single dollar coin). Make up a suitable generating function for this. Which coefficient are we looking for? Evaluate that coefficient using Maple (using the command `expand`).

Problem # 3

Determine whether the following sequences are graphical. If so, construct a graph with the appropriate degree sequence.

- (a) (4, 4, 3, 2, 1)
- (b) (3, 3, 2, 2, 2, 1, 1)
- (c) (7, 7, 6, 5, 4, 4, 3, 2)
- (d) (7, 6, 6, 5, 4, 3, 2, 1)
- (e) (7, 4, 3, 3, 2, 2, 2, 1, 1, 1)

Problem # 4

Show that the sequence (d_1, d_2, \dots, d_n) is graphical if and only if the sequence $(n - d_1 - 1, n - d_2 - 1, \dots, n - d_n - 1)$ is graphical.

Problem # 5

Verify that (3, 3, 2, 2, 1, 1, 1, 1) is the degree sequence of a tree. Construct three nonisomorphic trees with this degree sequences.

Problem # 6

Construct two different 4-regular graphs on seven vertices. Then prove that they are nonisomorphic.

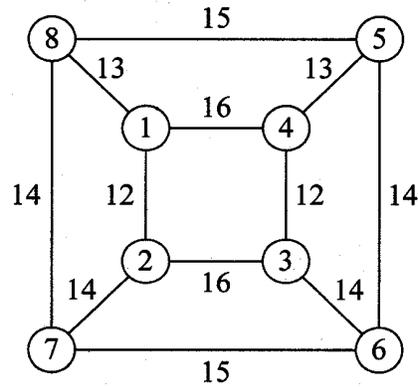
Problem # 7

Suppose that an urn contains four balls with different colors. In how many ways can three balls be chosen (explain) ?

	order important	order unimportant
repetition allowed		
repetition not allowed		

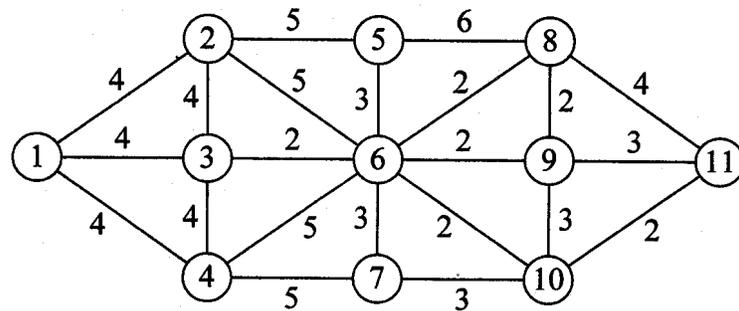
Problem # 8

Compute a minimum spanning tree for the following network using Prim's algorithm.



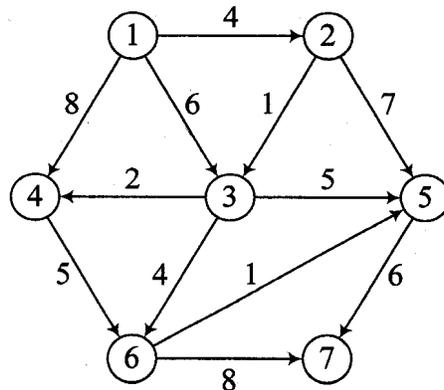
Problem # 9

Compute a minimum spanning tree for the following network using Prim's algorithm.



Problem # 10

Using Dijkstra's algorithm, compute the distances from vertex 1 in the following graph.



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