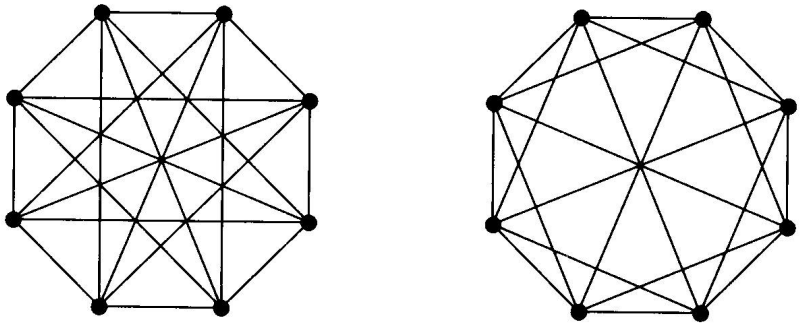
Dr. A. Betten Fall 2006

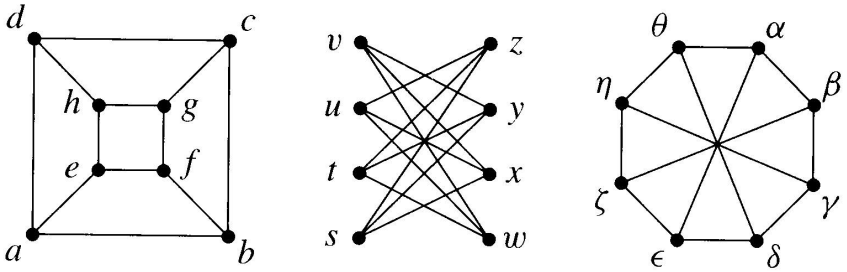
M301 Introduction to Combinatorial Theory

homework sheet # 4

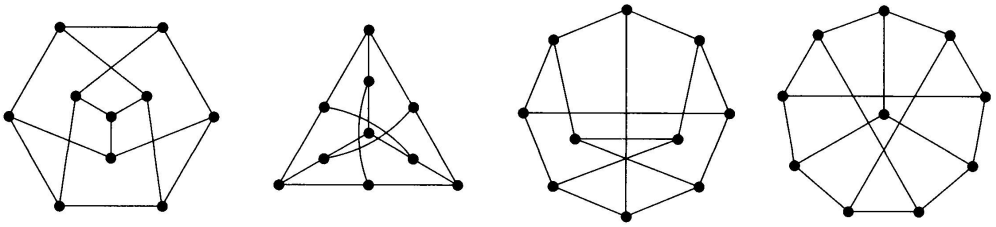
Problem # 1
Determine whether the graphs below are isomorphic.



Problem # 2
Determine which pairs of graphs below are isomorphic.

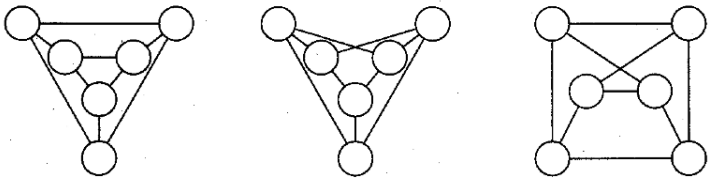


Problem # 3
Show that the graphs below are all isomorphic to the Petersen graph.

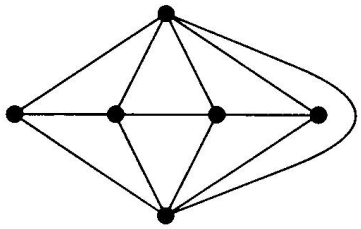


Problem # 4
Construct two nonisomorphic graphs with degree sequence $(1, 1, 2, 2, 3, 3)$.

Problem # 5
Determine which of the graphs below are isomorphic.

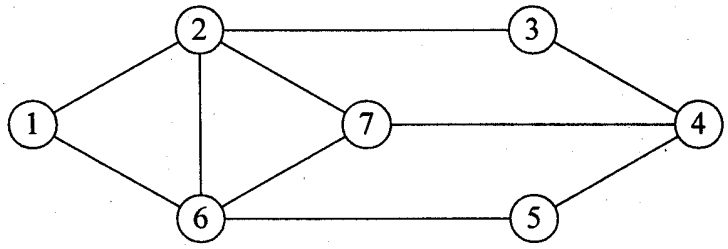


Problem # 6
In a graph $G = (V, E)$, a set I of vertices if said to form a *clique* if any two vertices of I are adjacent. A set I of vertices if called *independent* (or coclique) if no two vertices of I are adjacent. Determine the maximum size of a clique and the maximum size of an independent set in the graph below.



Problem # 7
A set K of vertices of G is caled a *vertex cover* if every edge of G is incident (i.e. adjacent) to at least one vertex in K. Show that a set K of vertices is a vertex cover if and only if its complement $V - K$ is an independent set.

Problem # 8
The *independence number* of a graph G, denoted $\alpha(G)$, is the size of a largest independent set in G. The *vertex covering number* of a graph G, denoted $\beta(G)$, is the size of a smallest vertex cover of G. Determine $\alpha(G)$ and $\beta(G)$ for the graph G below. Exhibit a largest independent set and a smallest vertex cover.



Problem # 9
Show that $\alpha(G) + \beta(G) = n$ for every simple graph G with n vertices.

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