

M301 Introduction to Combinatorial Theory

homework sheet # 3

Problem # 1

- (a) How many different five-digit numbers are there (leading zeros, e.g., 00174, not allowed)?
- (b) How many even five-digit numbers are there?
- (c) How many five-digit numbers are there with exactly one 3?
- (d) How many five-digit numbers are there that are the same when the order of the digits in inverted (e.g., 15251)?

Problem # 2

A rumor is spread randomly among a group of 10 people by successively having one person call someone who calls someone, and so on. A person can pass the rumour on to anyone except the individual who just called.

- (a) How many different paths can a rumor travel through the group in three calls? In n calls?
- (b) What is the probability that if A starts the rumor, A receives the thrid call?
- (c) What is the probability that if A does not start the rumor, A receives the third call?

Problem # 3

For each of the following expressions, list the set of all formal products in which the exponents sum to 4.

- (a) $(1 + x + x^2)^2(1 + x)^2$
- (b) $(1 + x + x^2 + x^3 + x^4)^3$
- (c) $(1 + x^2 + x^4)^2(1 + x + x^2)^2$
- (d) $(1 + x + x^2 + x^3 + \dots)^3$

Problem # 4

Build a generating function for a_r , the number of integer solutions to the equations:

- (a) $e_1 + e_2 + e_3 + e_4 + e_5 = r, 0 \leq e_i \leq 4$
- (b) $e_1 + e_2 + e_3 = r, 0 < e_i < 4$
- (c) $e_1 + e_2 + e_3 + e_4 = r, 2 \leq e_i \leq 8, e_1$ odd, e_2 even
- (d) $e_1 + e_2 + e_3 + e_4 = r, 0 \leq e_i$
- (e) $e_1 + e_2 + e_3 + e_4 = r, 0 < e_i, e_2, e_4$ odd, $e_4 \leq 3$

Problem # 5

Build a generating function for a_r , the number of r selections from a pile of:

- (a) Three red, four black, and four white balles.
- (b) Five jelly beans, three licorice sticks, eight lollipops with at least one of each candy.
- (c) Unlimited amounts of pennies, nickels, dimes, and quaters.
- (d) Seven types of lightbulbs with an odd number of the first and second types.

Problem # 6

Build a generating function for a_r , the number of distributions of r identical objects into:

- (a) Five different boxes with at most four objects in each box.
- (b) Four different boxes with between three and eight objects in each box.
- (c) Seven different boxes with at least one object in each box.
- (d) Three different boxes with at most five objects in the first box.

Problem # 7

Use a generating function for modeling the number of 5-combinations of the letters M, A, T, H in which M and A can appear any number of times but T and H at most once. Which coefficient in this generating function do we want?

Problem # 8

Find a generating function for the number of integers between 0 and 999,999 whose sum of digits is r .

Problem # 9

Explain why $(1 + x + x^2 + \cdots + x^r)^4$ is not a proper generating function for a_r , the number of ways to select r objects from four types with repetition. What is the correct generating function?

Problem # 10

Find the coefficient of x^8 in $(1 + x + x^2 + x^3 + \cdots)^n$.

Problem # 11

Find the coefficient of x^r in $(x^5 + x^6 + x^7 + \cdots)^8$.

Problem # 12

Find the coefficient of x^9 in $(1 + x^2 + x^4)(1 + x)^m$.

Problem # 13

How many ways are there to get a sum of 25 when 10 distinct dice are rolled?

Problem # 14

How many ways are there to select 240 chocolate candies from seven types if each type comes in boxes of 20 and if at least one but no more than five boxes of each type are chosen? (*Hint:* Solve in terms of boxes of chocolate.)

not collected