

# M301 Introduction to Combinatorial Theory

## homework sheet # 2

### Problem # 1

A circular permutation is an arrangement of distinct objects around a circle. Find the number of circular permutations of  $n$  distinct objects.

### Problem # 2

In how many ways can  $n$  couples be seated at a dinner table so that men and women alternate?

### Problem # 3

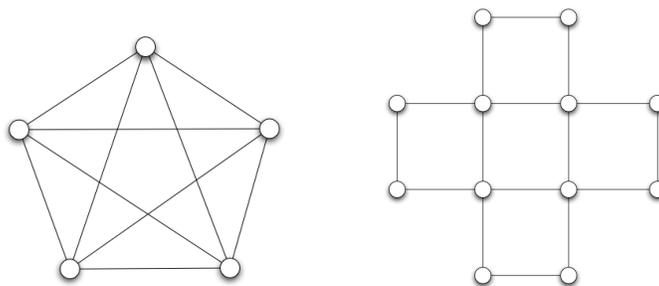
A library has  $n$  books. Readers of this library are “almost” careful. That is, after reading a book, they put it back to its shelf, missing its proper place by only one notch. Prove that after a sufficient amount of time (without any interaction by a librarian), any permutation of the books on the shelves can occur.

### Problem # 4

(a) An Euler cycle is a path through all the edges of a graph which starts and ends at the same vertex (you may visit the same vertex repeatedly but never go the same edge twice).

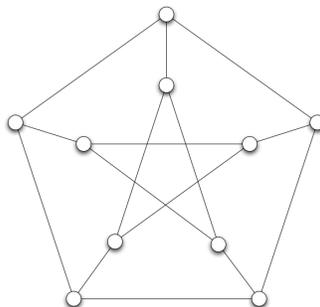
How many Euler cycles do each of the following graphs have?

(b) A Hamiltonian cycle is a cycle through all the vertices of the graph (you must visit every vertex exactly once). How many Hamiltonian cycles do each of the graphs have?



### Problem # 5

How many 6-cycles does Petersen's graph have?



**Problem # 6**

The zebra problem: There are five houses in a row, each of a different color, inhabited by women of different nationalities. The owner of each house owns a different pet, serves different drinks, and smokes different cigarettes from the other owners. The following facts are also known:

The Englishwoman lives in the red house.

The Spaniard owns a dog.

Coffee is drunk in the green house.

The Ukrainian drinks tea.

The green house is immediately to the right of the ivory house.

The Oldgold smoker owns the snail.

Kools are smoked in the yellow house.

Milk is drunk in the middle house.

The Norwegian lives in the first house on the left.

The Chesterfield smoker lives next to the fox owner.

The yellow house is next to the horse owner.

The Lucky Strike smoker drinks orange juice.

The Japanese smokes Parliament.

The Norwegian lives next to the blue house.

The question: Who drinks water and who owns the zebra?

**Problem # 7**

A group of ten children want to play cards. They split into three groups, one of these groups has four children in it, the other two have three each. Then each group sits around a table. Two seatings are considered the same if everyone's left neighbor is the same.

(a) In how many ways can this be done if the three tables are identical?

(b) In how many ways can this be done if the three tables are distinct?

**Problem # 8**

Show that  $\sum_{i=1}^n i(i+1) = \frac{n(n+1)(n+2)}{3}$ .

**Problem # 9**

Show that  $1^2 + 3^2 + 5^2 + 7^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ .

**Problem # 10**

Show that  $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$ .

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