This problem gives a nice illustration of a volume integral. At first glance, we don’t know enough to solve the problem, because we don’t know the diameter of the hole that was drilled. However, the rhyme implies we don’t need to know, so let’s try to solve it without that knowledge.

Here’s a drawing of the situation, in cross-section:

![Diagram of the situation](image)

We know that the total length of the hole is 6", so the length from the center to one end of the hole is 3". We can define the radius of the sphere as \( r \), and the unknown radius of the hole as \( a \).

What we need to find is the volume swept out by the shaded region as it is rotated around the axis of the hole. A stock formula for this type of volume is probably available in a reference book somewhere, but let’s work it out anyway.

Computing volume is a good application for an integral. In this problem, we have symmetry about the axis of the center of the hole, so it makes sense to define cylindrical shells centered on that axis, and integrate over them.

Our integral will run from \( a \) to \( r \) as the cylinders go from the inner surface of the hole to the rim of the sphere furthest from the hole axis. Let’s use \( x \) as the variable of integration, and let it run from \( a \) to \( r \). For a particular value of \( x \), the corresponding cylinder will have radius \( x \), and some length. The length will be based on the right triangle formed by the \( r \) and \( x \), and will be:

\[
2\sqrt{r^2 - x^2}
\]

Since the circumference for radius \( x \) is \( 2\pi x \), the area of the cylinder will be

\[
4\pi x \sqrt{r^2 - x^2}
\]

So, now we have our integral for the enclosed volume:

\[
v = \int_a^r \frac{1}{2} \pi x \sqrt{r^2 - x^2} \, dx
\]

Luckily, we have the derivative of the expression inside the radical as part of what’s outside the radical, so we can bring some constants outside the integral and get:

\[
v = -2\pi \left[ \frac{2}{3} \left( r^2 - x^2 \right)^{3/2} \right]_a^r = \frac{4\pi}{3} \left( r^2 - a^2 \right)^{3/2}
\]

But from our drawing, we know that for a given \( a \), we can calculate the value of \( r \) that makes the hole exactly 6 inches long, namely:

\[
r^2 = a^2 + 3^2
\]

Substituting this back into our volume relation gives

\[
v = \frac{4\pi}{3} \left( a^2 + 3^2 - a^2 \right)^{3/2} = \frac{4\pi}{3} (3^2)^{3/2} = 36\pi
\]

So the volume of the remaining portion of the sphere is not dependent on \( a \), and the rhyme was correct.