The point of this lab is to implement a few different versions of approximate integration. Really, they are just different versions of Riemann sums – there are more sophisticated ideas that we won’t touch. Before we begin the lab, recall from univariate calculus that you can approximate the area under a curve by drawing certain boxes. Dan will draw a picture for you....

The question is how tall to make the box. You could go with the right endpoint of each subinterval, the left endpoint, the midpoint, or a line between the left and right endpoints. (Again, there are more sophisticated options, like Simpson’s method.)

1. (3 points) Write a procedure “approxInt” that takes in a function \( f \), a number of intervals \( N \), a left endpoint \( a \), and a right endpoint \( b \). Given these data, the procedure should split the interval \([a, b]\) into \( N \) subintervals and use the function \( f \) to estimate the area under the curve using left endpoints. Have the procedure return the approximate area. Test it with one of the functions in the #5 below (or one of your own functions).

2. (2 points) Add a few lines to your procedure to also estimate the area using midpoints, right endpoints, and trapezoids. The procedure should now return four estimates.

3. (2 points) Write another function “compareApproxInt” that just takes in the function \( f \) and the endpoints \( a \) and \( b \). Have this function call “approxInt” five times, for \( N = 5, 10, 20, 40, 80 \). After each call, somehow store the four return values (approximate areas). After making those five calls, have Maple spit out a point plot that plots the behavior of each method in a different color (red for left endpoints, blue for right, etc. – pick your own colors!). In other words, plot the points \([N, \text{approxInt}]\) for each \( N \) and each of the four approximations per \( N \). To make the plot nicer, make sure the points are easily visible and also plot the origin in black.

4. (1 point) Also include in your graph a plot of the actual area, using the Maple command \texttt{int}.

5. (2 points) Test “compareApproxInt” with the functions \( x, -x, x^2, \) and \( -x^2 \), each on the interval \([-1, 1]\).

What to turn in: Your final two procedures and your tests.