1. Consider the system of equations

\[ \begin{align*}
  x + 2y &= -2 \\
 2x - y &= 6
\end{align*} \]

(a) Find all solutions to this system.

(b) On one copy of the \( x - y \) plane, graph the lines described by each of these two equations. If additional equations arise in the course of your derivation, graph them, too. Label the solution(s) to the system of equations.

2. Five cattle and two sheep cost ten liang of silver. Two cattle and five sheep cost five liang of silver. Tell: what is the cost of a cow? Of a sheep?

Nine Chapters on the Mathematical Art, Chapter 8, problem 7.

3. The goal of this exercise is to prove that the equation operation of adding a nonzero multiple of one equation to another equation results in an equivalent system (part 3 of Theorem EOPSS). More specifically, suppose you have three linear equations in three variables

\[ \begin{align*}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3.
\end{align*} \]

Prove that adding a multiple \( \alpha \neq 0 \) of equation 2 to equation 1 results in an equivalent linear system. To be more explicit:

(a) Suppose \((s_1, s_2, s_3)\) is a solution of the system above. Argue why it must also be a solution of

\[ \begin{align*}
  (a_{11}x_1 + a_{12}x_2 + a_{13}x_3) + \alpha(a_{21}x_1 + a_{22}x_2 + a_{23}x_3) &= b_1 + \alpha b_2 \\
  a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
  a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3.
\end{align*} \]

That shows that solutions of the original SLE must be solutions of the modified SLE.

(b) Now go the other way. Argue that a solution \((s_1, s_2, s_3)\) of the modified SLE must also be a solution of the original SLE.
4. Consider the linear system

\[
\begin{align*}
{x} + 2{y} - 2{z} &= -3 \\
-2{x} - 3{y} + 3{z} &= 3 \\
2{x} + 6{y} - 5{z} &= -3
\end{align*}
\]

(a) Use equation operations (not matrices) to find any solutions of this problem.
(b) Write down the corresponding augmented matrix.
(c) Put the augmented matrix into reduced row echelon form (RREF).
(d) Use your answer to part (c) to report any solutions of the original linear system. (You should get the same answer as in the part (a).)

5. Consider the linear system

\[
\begin{align*}
{w} + 2{x} + {y} + 9{z} &= 2 \\
-{w} - {x} - 4{z} &= -3 \\
{w} + {x} + {y} + 8{z} &= 1
\end{align*}
\]

(a) Use equation operations (not matrices) to find any solutions of this problem.
(b) Write down the corresponding augmented matrix.
(c) Put the augmented matrix into reduced row echelon form (RREF).
(d) Use your answer to part (c) to report any solutions of the original linear system. (You should get the same answer as in the part (a).)

6. Consider the linear system

\[
\begin{align*}
{x} + {y} + 5{z} &= 1 \\
-2{x} - {y} - 8{z} &= -3 \\
{x} - {y} + {z} &= 4
\end{align*}
\]

(a) Use equation operations (not matrices) to find any solutions of this problem.
(b) Write down the corresponding augmented matrix.
(c) Put the augmented matrix into reduced row echelon form (RREF).
(d) Use your answer to part (c) to report any solutions of the original linear system. (You should get the same answer as in the part (a).)

7. Which of the following five matrices are in reduced row echelon form (RREF)?

(a) \[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 3
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
1 & 0 & 0 & 2 & 3 \\
0 & 0 & 1 & 3 & 2 \\
0 & 0 & 0 & 4 & -6
\end{pmatrix}
\]

(c) \[
\begin{pmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{pmatrix}
\]

(d) \[
\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

(e) \[
\begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 1
\end{pmatrix}
\]