M229 Matrices and Linear Equations
Second Examination
Version SAMPLE

(8 pts) 1.  a) Find all (complex) zeros of \( f(x) = x^2 - 4x + 13 \).
b) Find the absolute value and argument of each answer to a).
c) Plot the answers to a) in the geometric representation.

(9 pts) 2.  a) Draw a digraph having \( A \) as adjacency matrix.
b) Write a formula that uses matrix multiplication
   and counts walks from vertex 1 to vertex 2 of
   length 10 in the digraph.
c) Compute the total number of walks of length 3 in
   the digraph.

(6 pts) 3.  Let \( A = \begin{pmatrix} 7 & -8 \\ 4 & -5 \end{pmatrix} \) and \( v = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). Suppose \( f(x) = f_0 + f_1x + x^2 \).
   a) Write the auxiliary matrix necessary to compute \( f(A)v \).
b) Solve for \( f(x) : f(A)v = 0 \)

(6 pts) 3.  Compute the \( A^{-1} \) where

\[
A = \begin{pmatrix} 3 & 0 & -4 \\ -2 & -1 & c \\ 2 & 0 & -3 \end{pmatrix}
\]

**WARNING** \( c \) is an arbitrary constant.
(8 pts) 4. Suppose $A$ is a $4 \times 2$ matrix.

a) Give the formula for the symmetric projector $P_A$.
b) State any conditions that $A$ must satisfy for this formula to be valid.
c) Use the formula from a) and properties of matrix multiplication to show that $P_A^2 = P_A$.

(8 pts) 4. Let $a = (1 \ - \ 1 \ 2)^T$, $v = (1 \ 5 \ -1)^T$

a) Give the formula for and compute the symmetric projector $P_{a^\perp}$.
b) Write $v$ as a linear combination of a scalar multiple of $a$ and a vector perpendicular to $a$.

(8 pts) 5. Four data points in the figure are given:
a) Find and sketch the degree 2 least squares approximation $f(x) = f_0 + f_1 x + f_2 x^2$ to the data.
b) Give the coordinates of the point in the sample space associated with $f(x)$

(10 pts) 6. Two plane coordinate systems are related as in the figure.
a) Write the (homogeneous) transition matrix $T_{(C \rightarrow C)}$ from the unprimed to the primed coordinates.
b) Compute the primed coordinates of the indicated point $P$.