We will not answer questions about this page during the exam.

\[ f \text{ is a real-valued function and } \mathbf{F}(x, y, z) = \langle M, N, P \rangle \text{ is vector-valued. (If in } \mathbb{R}^2, \mathbf{F} = \langle M, N \rangle. \rangle \]

\[ \mathbf{T} \text{ is an appropriate unit tangent vector and } \mathbf{n} \text{ is an appropriate unit normal vector.} \]

\[ \mathbf{r}(t) = (f(t), g(t), h(t)) \text{ is a parameterization of a curve in } \mathbb{R}^3 \text{ (} \mathbf{r}(t) = (f(t), g(t)) \text{ in } \mathbb{R}^2) \}; \]

\[ \mathbf{r}(u, v) = (f(u, v), g(u, v), h(u, v)) \text{ is a parameterization of a surface, with } \mathbf{r}_u = \frac{\partial \mathbf{r}}{\partial u} \text{ and } \mathbf{r}_v = \frac{\partial \mathbf{r}}{\partial v}. \]

\[ \text{Line Integral along a curve } C : \quad \int_C f(x, y, z) \, ds = \int_a^b f(\mathbf{r}(t))|\mathbf{v}(t)| \, dt, \text{ where } \mathbf{v}(t) = \mathbf{r}'(t). \]

\[ \text{Work/Circ/Flow along a curve } C : \]

in \( \mathbb{R}^2 \): Work/Circ/Flow = \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy. \) Also see Green’s Theorem.

in \( \mathbb{R}^3 \): Work/Circ/Flow = \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy + P \, dz. \) Also see Stokes’ Theorem.

Flux of vector field \( \mathbf{F} \):

across curve \( C \subset \mathbb{R}^2 \): \( \int_C \mathbf{F} \cdot \mathbf{n} \, ds = \int_C M \, dy - N \, dx. \) Also see Green’s Theorem.

through surface \( S \subset \mathbb{R}^3 \): \( \int_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \int_S \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, du \, dv. \) Also see Divergence Theorem.

Component Test: \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}. \)

Fundamental Theorem for Line Integrals: If \( \mathbf{F} = \nabla f \) and curve \( C \) goes from \( A \) to \( B \), then

\[ \int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A). \]

Green’s Theorem: Region \( R \subset \mathbb{R}^2 \) has closed boundary curve \( C \).

Work/Circ/Flow = \( \oint_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dx \, dy = \iint_R \nabla \times \mathbf{F} \cdot \mathbf{k} \, dx \, dy, \)

Flux = \( \int_C \mathbf{F} \cdot \mathbf{n} \, ds = \int_C M \, dy - N \, dx = \iint_R \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \, dx \, dy = \iint_R \nabla \cdot \mathbf{F} \, dx \, dy \)

Surface Integral of \( g \) over the surface \( S \) (\( g(x, y, z) = 1 \) for surface area):

\[ \iint_S g(x, y, z) \, d\sigma = \iint_R g(\mathbf{r}(u, v))|\mathbf{r}_u \times \mathbf{r}_v| \, du \, dv, \text{ with parameters } u, v \text{ in } R. \]

Stokes’ Theorem: Surface \( S \) with closed boundary curve \( C \) (where \( C \) has counterclockwise orientation with respect to the normal direction of \( S \)).

Work/Circ/Flow = \( \oint_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy + P \, dz = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iint_S \nabla \times \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, du \, dv. \)

Divergence Theorem: Solid \( D \) with boundary surface \( S \).

\[ \text{Flux} = \iiint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iint_S \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, du \, dv = \iiint_D \nabla \cdot \mathbf{F} \, dV = \iiint_D \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} \, dV. \]