Math 261 Exam 3 Review Formulas & Reminders
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Here are a few formulas that might be handy for Exam 3. You cannot bring this to the exam, but hopefully it helps with studying....

WARNING: I do not guarantee that this is a comprehensive list! Also, please note that there are various alternative formulations for some of these formulas – I am just picking those that I like the best. Finally, there could be typos – beware!

• \( \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx \) determines the volume below the graph of the function \( f(x, y) \) and above the \( xy \)-plane. For each \( x \) between \( a \) and \( b \), \( y \) runs from \( g_1(x) \) to \( g_2(x) \). The value of this integral is the same as \( \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx \, dy \) for appropriate choices of \( h_1(y), h_2(y), c, d \). Know how to switch between the two, how to sketch the region given by some bounds, and how to read the bounds off of a sketch.

• The area of a region in the plane is just the integral of the function \( f(x, y) = 1 \) over that region. The average function value over a region is the volume under the graph over the region divided by the area of the region.

• Polar coordinates: Know how to set up/do double integrals using polar coordinates.

• Triple integrals in rectangular – Know how to set up integrals in any variable order. The bounds are the hard part here. Integrate 1 for volume.

• Mass, moments: Know the formulas for these (mass & first moments only, not second moments) and how to spot that some coordinate of the center of mass is 0 (by symmetry).

• Cylindrical, spherical: Know how to set up/do triple integrals in these coordinate systems. Depending on the shape, you might need to do 2 triple integrals to cover it all.

• Substitution in multiple integrals: Know how to find the determinant of the Jacobian of a substitution & don’t forget to plug in the absolute value of it when changing variables. Know how to figure out the new bounds from the old ones. There will not be any 3-variable substitution problems on the exam.

• Line integrals of scalar functions: Know how to set up and do these, along with how to parameterize a line segment and a circle (e.g., \( \langle \cos t, \sin t \rangle \)). There is a short section on mass & moments of wires & strips in this section, too.

• Vector fields & line integrals: Know how to set up a line integral for a curve through a vector field \( \text{work} = \text{circ} = \text{flow} \) and how to set up the line integral for flux.

The main things we are excluding from what we have taught you are 3-variable substitution, the expectation that you draw complicated sketches (know how to handle spheres, cylinders, circles, lines, planes, and such; not quadric surfaces), and long, complicated integrals. The best way to prepare is to check out the three old exams on the website and try problems like those. I am changing Exam 3 some this year, but those old exams are still good preparation (just like climbing Mt. Everest is good preparation for climbing Longs Peak).