**We will not answer questions about this page during the exam.**

\( f \) is a real-valued function and \( \mathbf{F}(x, y, z) = \langle M, N, P \rangle \) is vector-valued. (If in \( \mathbb{R}^2 \), \( \mathbf{F} = \langle M, N \rangle \).

\( \mathbf{T} \) is an appropriate unit tangent vector and \( \mathbf{n} \) is an appropriate unit normal vector.

\( \mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle \) is a parameterization of a curve in \( \mathbb{R}^3 \) \( \mathbf{r}(t) = \langle f(t), g(t) \rangle \) in \( \mathbb{R}^2 \);

\( \mathbf{r}(u, v) = \langle f(u, v), g(u, v), h(u, v) \rangle \) is a parameterization of a surface, with \( r_u = \frac{\partial \mathbf{r}}{\partial u} \) and \( r_v = \frac{\partial \mathbf{r}}{\partial v} \).

**Line Integral along a curve** \( C \):
\[ \int_C f(x, y, z) \, ds = \int_a^b f(\mathbf{r}(t))|\mathbf{v}(t)| \, dt, \text{ where } \mathbf{v}(t) = \mathbf{r}'(t). \]

**Work/Circ/Flow along a curve** \( C \):
- in \( \mathbb{R}^2 \): Work/Circ/Flow = \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy \). Also see Green’s Theorem.
- in \( \mathbb{R}^3 \): Work/Circ/Flow = \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy + P \, dz \). Also see Stokes’ Theorem.

**Flux of vector field** \( \mathbf{F} \):
- across curve \( C \subset \mathbb{R}^2 \): \( \int_C \mathbf{F} \cdot \mathbf{n} \, ds = \int_C M \, dy - N \, dx \). Also see Green’s Theorem.
- through surface \( S \subset \mathbb{R}^3 \): \( \iint_S \mathbf{F} \cdot \mathbf{n} \, d\mathbf{\sigma} = \iint_S \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dudv \). Also see Divergence Theorem.

**Component Test:** \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} \).

**Fundamental Theorem for Line Integrals:** \( \mathbf{F} = \nabla f \), curve \( C \) goes from \( A \) to \( B \).
\[ \int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A). \]

**Green’s Theorem:** Region \( R \subset \mathbb{R}^2 \) has closed boundary curve \( C \).

**Work/Circ/Flow** = \( \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dx \, dy = \iint_R \nabla \times \mathbf{F} \cdot \mathbf{k} \, dx \, dy \),
\[ \text{Flux} = \oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R M \, dy - N \, dx = \iint_R \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \, dx \, dy = \iint_R \nabla \cdot \mathbf{F} \, dx \, dy \]

**Surface Integral of** \( g \) over the surface \( S \) \((g(x, y, z) = 1 \text{ for surface area}): \)
\[ \iiint_S g(x, y, z) \, d\mathbf{\sigma} = \iint_R g(\mathbf{r}(u, v))|\mathbf{r}_u \times \mathbf{r}_v| \, du \, dv, \text{ with parameters } u, v \text{ in } R. \]

**Stokes’s Theorem:** Surface \( S \) with closed boundary curve \( C \).

**Work/Circ/Flow** = \( \iint_S \mathbf{F} \cdot d\mathbf{\sigma} = \iint_S \mathbf{F} \cdot \mathbf{n} \, d\mathbf{\sigma} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, d\mathbf{\sigma} = \iint_S \nabla \times \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dudv. \)

**Divergence Theorem:** Solid \( D \) with boundary surface \( S \).
\[ \text{Flux} = \iint_S \mathbf{F} \cdot \mathbf{n} \, d\mathbf{\sigma} = \iiint_D \nabla \cdot \mathbf{F} \, dV = \iiint_D \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} \, dV. \]