Here are a few formulas that might be handy for Exam 1. You cannot bring this to the exam, but hopefully it helps with studying....

**WARNING:** I do not guarantee that this is a comprehensive list! Also, please note that there are various alternative formulations for some of these formulas – I am just picking those that I like the best. Finally, there could be typos – beware!

- **(Vector between two points)** Given points \((p_1, p_2, p_3)\) and \((q_1, q_2, q_3)\) in \(\mathbb{R}^3\), the vector between them is just \(Q - P = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle\).
- **(Length of a vector)** \(|v| = |\langle v_1, v_2, v_3 \rangle| = \sqrt{v_1^2 + v_2^2 + v_3^2}\).
- **(Make a vector unit length)** Just divide the vector by its length: \(\frac{v}{|v|}\).
- **(Dot product)** \(\mathbf{v} \cdot \mathbf{w} = \langle v_1, v_2, v_3 \rangle \cdot \langle w_1, w_2, w_3 \rangle = v_1w_1 + v_2w_2 + v_3w_3\). (One way to get the normal is to take the cross product of \(\mathbf{v}\) and \(\mathbf{w}\).) Don’t forget to negate that middle coordinate! (It might be easier to remember the 3 Xs way I taught you to compute the cross product.) Recall that two nonzero vectors are parallel if and only if \(\mathbf{v} \parallel \mathbf{w}\) are orthogonal if and only if \(\mathbf{v} \cdot \mathbf{w} = 0\).
- **(Another dot product formula)** \(\mathbf{u} \cdot \mathbf{v} = |u||v| \cos \theta\), where \(\theta\) is the angle between \(\mathbf{u}\) and \(\mathbf{v}\).
- **(Cross product)** \(\mathbf{v} \times \mathbf{w} = \langle v_1, v_2, v_3 \rangle \times \langle w_1, w_2, w_3 \rangle = \langle v_2w_3 - v_3w_2, -v_1w_3 + v_3w_1, v_1w_2 - v_2w_1 \rangle\) (a vector!). Don’t forget to negate that middle coordinate! (It might be easier to remember the 3 Xs way I taught you to compute the cross product.) Recall that two nonzero vectors are parallel if and only if their cross product is the zero vector.
- **(Area of a triangle)** The area of a triangle with edges \(\mathbf{v}\) and \(\mathbf{w}\) is \(|\mathbf{v} \times \mathbf{w}|/2\). The volume of a parallelepiped with edges \(\mathbf{u}, \mathbf{v},\) and \(\mathbf{w}\) is \(|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|\).
- **(Another cross product formula)** \(\mathbf{v} \times \mathbf{w} = (|\mathbf{v}||\mathbf{w}| \sin \theta) \mathbf{n}\) where \(\theta\) is the angle between \(\mathbf{v}\) and \(\mathbf{w}\) and \(\mathbf{n}\) is a unit vector in the normal direction (orthogonal to \(\mathbf{v}\) and \(\mathbf{w}\)).
- **(Equations for a line)** Given a point \(P = (p_1, p_2, p_3)\) on a line and a vector \(\mathbf{v} = \langle v_1, v_2, v_3 \rangle\) in the direction of the line, the following are parametric equations for the line:

\[
\begin{align*}
x(t) &= p_1 + tv_1 \\
y(t) &= p_2 + tv_2 \\
z(t) &= p_3 + tv_3,
\end{align*}
\]

which could also be written more succinctly as \(\mathbf{r}(t) = \langle p_1 + tv_1, p_2 + tv_2, p_3 + tv_3 \rangle\). (Parameterizations are not unique - they depend on the choices of \(P\) and \(\mathbf{v}\).)
- **(Distance from point to line)** Given point \(S\) and a line with direction \(\mathbf{v}\) and a point (any point) \(P\), the distance from \(S\) to the line is \(\frac{|\mathbf{PS} \times \mathbf{v}|}{|\mathbf{v}|}\), where \(\mathbf{PS}\) is the vector from \(P\) to \(S\). If the line is given to you in parametric form, you can find a point on the line by plugging in any value of \(t\), e.g., \(t = 0\).
- **(Equation for a plane)** A plane is given by a normal vector \(\langle A, B, C \rangle\) and a point \((x_0, y_0, z_0)\) on the plane. The equation is then \(A(x - x_0) + B(y - y_0) + C(z - z_0) = 0\). One way to get the normal is to take the cross product of two vectors in the plane (that have the same initial point).
• (Line of intersection of two planes) Given two planes with normal vectors $n_1$ and $n_2$, respectively, the vector $n_1 \times n_2$ points in the direction of the line of intersection of the two planes (assuming they intersect!). To get a point on this line, you can solve the system of two plane equations. Note that there will be 3 variables and 2 equations in this linear system, so you should just set one of the variables to a constant, e.g., $x = 0$, and solve for the other two. (Two planes are parallel – don’t intersect – if they have parallel normals, i.e., $n_1 \times n_2 = 0$.)

• (Distance from point to plane) Given point $S$ and a plane with normal $n$ and point $P$ on the plane, the distance from $S$ to the plane is $\frac{PS \cdot n}{|n|}$.

• (Angle between planes) The angle between two planes is the angle between their normal vectors.

• (Position, velocity, and acceleration) If $r(t)$ represents the position of a particle, then $v(t) = r'(t)$ is the velocity and $a(t) = v'(t)$ is the acceleration. If you are given an initial value problem (e.g., $a(t)$ and the values of $v(t)$ and $r(t)$ at some point), you integrate to work your way up to $r(t)$, using the values of $v(t)$ and $r(t)$ to find the constants of integration.

• (Arc length) If $r(t) = \langle x(t), y(t), z(t) \rangle$ gives a curve in $\mathbb{R}^3$, the arc length from the point at time $t = a$ to the point at time $t = b$ is

\[
  s = \int_a^b |v(t)| \, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt.
\]

• (Decomposing acceleration) Acceleration along a curve can be written as a linear combination of the tangent and normal directions. In particular:

\[
a(t) = a_T(t)T(t) + a_N(t)N(t),
\]

where:

- $T(t) = \frac{v(t)}{|v(t)|}$,
- $N(t) = \frac{T'(t)}{|T'(t)|}$ (this is not the easy way to compute this!),
- $a_T = a \cdot T$, and
- $a_N$ can be found from the Pythagorean identity $|a|^2 = a_T^2 + a_N^2$.

If you need to compute all of these, the typical strategy is to compute $v$ and $a$, then compute $T$ as above, then $a_T$, then $a_N$. Finally, you can find $N$ using the fact that $a = a_T T + a_N N$. 