We will not answer questions about this page during the exam.

\(f\) is a real-valued function and \(\mathbf{F}(x, y, z) = \langle M, N, P \rangle\) is vector-valued. (If in \(\mathbb{R}^2\), \(\mathbf{F} = \langle M, N \rangle\).) \n
\(\mathbf{T}\) is an appropriate unit tangent vector and \(\mathbf{n}\) is an appropriate unit normal vector. \n
\(\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle\) is a parameterization of a curve in \(\mathbb{R}^3\) (\(\mathbf{r}(t) = \langle f(t), g(t) \rangle\) in \(\mathbb{R}^2\)); \n
\(\mathbf{r}(u, v) = \langle f(u, v), g(u, v), h(u, v) \rangle\) is a parameterization of a surface, with \(\mathbf{r}_u = \frac{\partial \mathbf{r}}{\partial u}\) and \(\mathbf{r}_v = \frac{\partial \mathbf{r}}{\partial v}\).

**Line Integral along a curve** \(C\): \n
\[
\int_C f(x, y, z) \, ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{v}(t)| \, dt, \text{ where } \mathbf{v}(t) = \mathbf{r}'(t).
\]

**Work/Circ/Flow along a curve** \(C\): \n
- in \(\mathbb{R}^2\): Work/Circ/Flow = \(\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy\). Also see Green’s Theorem. \n
- in \(\mathbb{R}^3\): Work/Circ/Flow = \(\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C M \, dx + N \, dy + P \, dz\). Also see Stokes’ Theorem.

**Flux of vector field** \(\mathbf{F}\): \n
across curve \(C \subset \mathbb{R}^2\): \(\int_C \mathbf{F} \cdot \mathbf{n} \, ds = \int_C M \, dy - N \, dx\). Also see Green’s Theorem. \n
through surface \(S \subset \mathbb{R}^3\): \(\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_R (\mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)) \, du \, dv\). Also see Divergence Theorem.

**Component Test:** \(\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}\).

**Fundamental Theorem for Line Integrals:** If \(\mathbf{F} = \nabla f\) and curve \(C\) goes from \(A\) to \(B\), then \(\int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A)\).

**Green’s Theorem:** Region \(R \subset \mathbb{R}^2\) has closed boundary curve \(C\). \n
Work/Circ/Flow = \(\iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} \, dx \, dy\), \n
Flux = \(\iint_R (\nabla \times \mathbf{F}) \cdot d\mathbf{F} \, dx \, dy\)

Surface Integral of \(g\) over the surface \(S\) (\(g(x, y, z) = 1\) for surface area): \(\iint_S g(x, y, z) \, d\sigma = \iint_R g(\mathbf{r}(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| \, du \, dv\), with parameters \(u, v\) in \(R\).

**Stokes’ Theorem:** Surface \(S\) with closed boundary curve \(C\). \n
Work/Circ/Flow = \(\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{F} = \iint_S (\mathbf{r}_u \times \mathbf{r}_v) \, du \, dv\).

**Divergence Theorem:** Solid \(D\) with boundary surface \(S\). \n
Flux = \(\iiint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_S (\mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)) \, du \, dv\) = \(\iint_D (\nabla \cdot \mathbf{F}) \, dV = \iint_D (\nabla \cdot \mathbf{F}) \, dV = \iint_D \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} \, dV\).