Math 261 Exam 2 Review Formulas & Reminders

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Here are a few formulas that might be handy for Exam 2. You *cannot* bring this to the exam, but hopefully it helps with studying....

WARNING: I do not guarantee that this is a comprehensive list! Also, please note that there are various alternative formulations for some of these formulas – I am just picking those that I like the best. Finally, there could be typos – beware!

- Basic limit computations
- Know how to find two paths with different limits to show that a limit doesn't exist (for functions with 2 or more variables).
- Multivariate chain rule: Draw the diagram of dependencies and trace from the function at the top to the desired variable at the bottom in each possible way.
- The derivative of the function f in the direction $u = \langle u_1, u_2, u_3 \rangle$ at the point $p = (p_1, p_2, p_3)$ is $D_u f(P) = \nabla f(P) \cdot \frac{u}{|u|}$. The direction of greatest increase is $u = \nabla f(P)$ – be sure to divide by the magnitude if a unit vector is needed.
- The equation for the plane tangent to the level surface $f(x, y, z) = a$ at point $p = (x_0, y_0, z_0)$ is $T(x, y, z) = f_x(p)(x - x_0) + f_y(p)(y - y_0) + f_z(p)(z - z_0)$. Similarly, the plane tangent to the graph of $z = f(x, y)$ at $p = (x_0, y_0)$ is $T(x, y, z) = f_x(p)(x - x_0) + f_y(p)(y - y_0) - (z - f(p))$.
- The linearization of $f(x, y)$ at $p = (x_0, y_0)$ is $L(x, y) = f(p) + f_x(p)(x - x_0) + f_y(p)(y - y_0)$.
- The error in linearizing at p over the region R is bounded as follows: $|E(x, y)| \leq \frac{M}{2}(|x - x_0| + |y - y_0|)^2$, where M is an upper bound on the absolute value of all second derivatives over the region R.
- To find the critical points of $f(x, y)$, set all first partials to 0 (i.e., $f_x = f_y = 0$) and find all solutions. Points of discontinuity are also critical points. For each critical point, compute $f_{xx} f_{yy} - f_{xy}^2$. If less than zero, this is a saddle point. If greater than 0, it is a local min (if $f_{xx} > 0$) or a local max (if $f_{xx} < 0$). If $f_{xx} f_{yy} - f_{xy}^2 = 0$, then the test is inconclusive.
- To find the absolute max and min over a bounded region R (given by inequality constraints), first find all critical points in the plane, discarding any not in R. Then find all critical points along each portion of the boundary (being sure to include all intersections between the various portions of the boundary, e.g., the points of a rectangle). Finally, for all points of interest, compute the function value and choose the max and the min.
- For Lagrange multipliers (problems with equality constraints), choose the objective function f and the constraint g. (f is sometimes $x^2 + y^2$ when you are seeking the point nearest the origin.) Solve the system of equations

  $$\nabla f = \lambda \nabla g$$

  $$g = 0$$

  for x, y, and λ. The max and min will be among these solutions. (Solving the system can be tricky. The best thing you can do to prepare is to try lots of problems.)