Problem 1 (Global optimization). Recall problem 2 from homework 5. There we considered the following mechanical system of bodies and springs:

The three springs at the top are suspended from the ceiling at positions \((x, z) = (-20\text{cm}, 0\text{cm}), (-5\text{cm}, 0\text{cm}), \text{ and } (7\text{cm}, 0\text{cm})\) and have rest lengths of \(L_{\text{top}} = 20\text{cm}\). The 4 bodies are connected with 4 springs of rest lengths \(L_{\text{between}} = 5\text{cm}\). All springs have a spring constant of \(D = 300 \frac{N}{m}\) and are attached to the centers of the bodies (which for the purpose of this problem we will consider as point-like). All bodies have a mass of \(m = 500g\).

We had argued in class on several occasions that there are likely many local minima, even though there is probably only a single global one.

Using one of the global optimization methods discussed in class (or any other one you may have come across), try to identify as many local minima as you can. For each, plot the positions of the four bodies and the springs that connect them; state the corresponding energy; and provide some kind of indication why you believe this to actually be a local minimum or at least be close to it.

There is no clear and obvious answer to how to achieve this goal. In particular, most global optimization methods provide a set of “samples” of which one typically just picks the best one (i.e., the one with the smallest objective function value) as an indication of the global minimum; however, these methods are not usually used to identify local minima. You will have to show some creativity and I will give some points for well-articulated and creative ideas to identify local minima from your computations, even if these ideas happen to not work well. Treat this as a mini research project. (15 points)

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!