MATH 689: Numerical Optimization

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Homework assignment 9 – due Tuesday 4/1/2014

Problem 1 (Optimality conditions, part 1) Consider the following problem:

$$\min_{x\in\mathbb{R}^2} \; x_1^2 + x_2$$
$$x_2 \geq 0.$$

Answer the following questions: (i) What is the solution? (ii) What are the necessary first order conditions for this problem? (iii) Verify that the solution indeed satisfies the first order conditions. (iv) What are the second order sufficient conditions for this problem? (v) Verify that the solution indeed satisfies the second order sufficient conditions. **(4 points)**

Problem 2 (Optimality conditions, part 2) Repeat the same steps as in problem 1 for the following version of the problem:

$$\min_{x\in\mathbb{R}^2} \; x_1^2 + x_2^2.$$
$$x_2 \geq 0.$$

What is the additional complexity with this problem? **(4 points)**

Problem 3 (Optimality conditions, part 3) Consider the following problem:

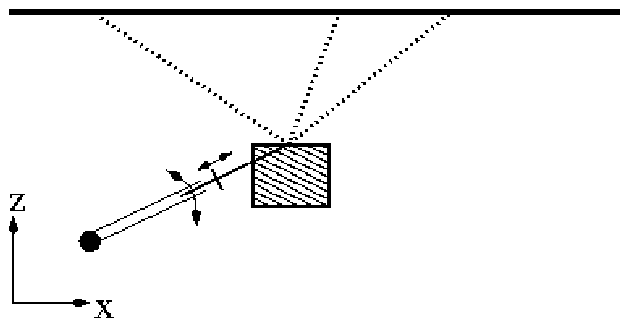
$$\min_{x\in\mathbb{R}^2} \; -\|x\|_{\ell_2}^2,$$
$$\|x\|_{\ell_\infty} \leq 1.$$

The objective function is not convex (in fact, it is concave) and the constraint is not differentiable. However, the problem can be reformulated as follows:

$$\min_{x\in\mathbb{R}^2} \; -\|x\|_{\ell_2}^2,$$
$$-x_1+1 \geq 0, \qquad x_1+1 \geq 0, \qquad -x_2+1 \geq 0, \qquad x_2+1 \geq 0.$$

Find all four (local) solutions of this problem. Verify for one of these points that the first and second order necessary conditions are satisfied at this point. **(4 points)**

Problem 4 (The same old problem, once more). Consider the following system of three springs suspended from the ceiling at positions $(x, z) = (-20\text{cm}, 0\text{cm})$, $(5\text{cm}, 0\text{cm})$ and $(15\text{cm}, 0\text{cm})$ and a rod of *minimal* length 35cm that is attached at one end at $(-25\text{cm}, -20\text{cm})$ and at the other end to the same point where the springs meet each other:



Each spring has a rest length of $L_0 = 20\text{cm}$, and extending (or compressing) spring $i, i = 1 \dots 3$ to a length L_i requires an energy of $E_{\text{spring},i} = \frac{1}{2}D(L_i - L_0)^2$ where the spring constant for all three springs equals $D = 300\frac{N}{m}$. On the other hand, the potential energy of the body is $E_{\text{pot}} = mgz$ where the body's mass is $m = 500g$, the gravity constant is $g = 9.81\frac{\text{m}}{\text{s}^2}$, and z is the vertical coordinate of the body's position.

Find the point of minimal energy using the active set SQP method. List how many iterations you need and provide a plot showing how the iterates converge.

(6 points)

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!