MATH 442: Mathematical Modeling

Fall 2014

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Office hours:	Thursdays, 2:00–4:00pm or by appointment
Lecture:	Tuesdays + Thursdays, 8:00–9:15am Blocker Bldg., Room 133

Writing assignment 2

Assignment

Describe a system that consists of 3 or more species that somehow interact with each other and derive a mathematical model for the populations of your species, as a function of time. The interactions can take the form of predation, competition for resources, symbiosis, toxicity, or any other way you can think of (not every species needs to interact with every other species). The species you consider may be plants, animals, bacteria, viruses, actors in a marketplace, snowflakes with sizes in different size groups, or anything else. If you have just a few species, you can name them individually, but if you have many it may be easier to just enumerate them as part of a food web.

You can invent your interacting system of species if you want, or you can use one that is described in the literature or on the internet. Obviously, if you use resources and data from elsewhere, you will need to reference these in your report.

Your report should have the following sections and subsections:

1. Introduction: State what you want to do and give an outlook on the rest of the report.
2. The model:
   - A discussion in words of the system you want to model.
   - A qualitative discussion of the factors that affect the dynamics of your populations.
   - An assessment which of these factors you think are important and consequently need to be represented in your mathematical model.
   - A quantitative discussion of population dynamics. This is where you derive and state a complete mathematical model in terms of differential equations.
3. Parameters: Discuss realistic values for all parameters that appear in your model and provide reasoning for why you think that these values are (at least roughly) correct. There are situations where parameters are only known to be in same (occasionally very large) range. In that case, state the range and a "most likely" value.
4. Predictions: Present numerical computations illustrating the long-term behavior of your system of populations. These should include:
   - visualizations of your population sizes over appropriately chosen time scales;
   - predictions that certain species will dominate or go extinct in the long run, or that the system oscillates like the solutions to the Lotka-Volterra equations;

- demonstrations how the dynamics change if you modified the values of the parameters or initial population sizes; for example, do certain parameters always lead to extinctions whereas others lead to stable states?

5. Analysis: Present a mathematical analysis of your system that discusses, if you can,
   - long-term asymptotic behavior;
   - existence of steady states;
   - stability of steady states;
   - non-negativity of population sizes.
6. Summary and conclusions.

There are no lower or upper bounds on your paper's length. However, you will need to address each of the points above in sufficient detail. (I anticipate that you will need 10–15 pages to do so, but this is just a guess and not a recommendation.)

Specifics

- This is a group project with groups of two. Both partners are responsible for working together on the project and will produce a single report. However, one partner will write the introduction, model, and parameters section. The other is responsible for writing up the computational and analytical results section, as well as the summary and conclusions.
- It must be clear from your report who wrote which part of it.
- You should peer review each other's sections.
- Write your report in the LyX word processor starting from the IEEEtran.lyx style file (or, if you want and are familiar with it, in straight LaTeX, for example using the `kile` editor).
- Submit a draft for feedback by 10/9/2014 8a.m. Email it to me before this date (`bangerth@math.tamu.edu`).
- The final version of your assignment is due Friday 10/17/2014 5p.m. You need to submit it before this date through the eCampus "Turninit Assignments by Groups" function.

Grading

Grading will be based on the following rubric:

- *Sophistication of model and its description 40%:* Your model should be a nontrivial case of a model. For example, a three-species model where species B eats A and C eats B is not very interesting. This is a straightforward extension of the Lotka-Volterra system. On the other hand, if B eats A, C eats B, A is constrained by a limited amount of food, and when C dies naturally it serves as food for fungus species D that is in turn eaten by A, then it becomes interesting. Your explanation of the system you have in mind should be complete and easy to comprehend.

  This part of the grading is done for the entire project and therefore applies to every group member.
- *Creativity 10%:* Be creative in how you choose your system. Don't just copy something you find in an existing paper. It is not necessary that the system you describe actually exists in nature and that the parameters you choose are realistic – you can be whimsical, but you need to be able to motivate and explain your choices. In practice, it is often easy to derive models in which every species, or every species but one, goes extinct. These are not very interesting from a practical point of view. Try to find models and parameters that show more interesting dynamics.

  This part of the grading is done for the entire project and therefore applies to every group member.

- *Completeness 30%:* The part of the paper that is your responsibility should be complete, easy to understand, written for someone at your level of knowledge regarding mathematical models (e.g., a class mate), and not have missing information or missing connections in a train of thought. You will also need to address the points outlined above.

  This part of the grading is done for the two parts of the report individually and only applies to the author of this particular part.

- *Structure, style and clarity 20%:* As a text, your report should be easy to read, i.e., it should be written with a reasonable subdivision into subsections, in proper English, and at an appropriate level of technicality suitable for your audience. It should include formulas where appropriate, show Maple commands if necessary for someone like you to repeat a computation, should have axes in graphs appropriately labeled, etc. Put yourself in the shoes of your reader if you think about whether to include a detail or not.

  This part of the grading is done for the two parts of the report individually and only applies to the author of this particular part.

Why I'm assigning this project

Mathematical modeling is, at its core, about seeing something in nature, describing what factors affect it, putting it all into formulas to obtain a *mathematical* model, and then analyzing what you have. Whether you knew it or not, you have used such models for many years already – in your calculus classes, in particular in your ODE class and probably the science classes you took, as well as in statistics. This is your opportunity to go through the process of coming up with a model yourself, see what it takes to completely describe and analyze it, and then to write it up in a report for your peers.
This assignment is designed to mimick what many of you will have to do at work in the not too distant future: take a system (maybe populations, or maybe some device you're asked to engineer, or a financial instrument), describe it in layman's language, derive a way to model it mathematically, and then provide predictions and anlyses for it. All of this then needs to be packaged so that your co-workers who will inherit the project, as well as those above you who have to make decisions based on your report, can understand what you were doing and why, and if necessary reproduce your results.