MATH 652: Optimization II

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Homework assignment 9 – due Thursday 4/15/2010

Problem 1 (An optimal control problem). In class, we had the example of a rover driving from x_A to x_B in a landscape in which the velocity attainable at position x given a force $q(t)$ was given by $\phi(x(t))q(t)$. The associated optimal control problem of finding the path of least energy consumption was given by

$$\min_{q(t)} \frac{1}{2}\int_0^T q(t)^2 \mathrm{d}t$$
$$\dot{x}(t) = \phi(x(t))q(t),$$
$$x(0) = x_A,$$
$$x(T) = x_B.$$

Find the optimal control $q(t)$ and associated optimal path $x(t)$ for the following conditions:

$$x_A = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \qquad T = 1,$$
$$x_B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \phi = \begin{cases} 1 & \text{if } x_2 < 0 \\ 2 & \text{if } x_2 \geq 0 \end{cases}.$$

Finding the solution to this problem is not easy if you don't know where to look. One way is to conjecture that the path will consist of two straight pieces from x_A to $(\alpha, 0)^T$ to x_B with some α, and that $q(t)$ is constant on each of these segments. Using this assumption, you can put a trial solution into the optimality conditions to see if you can satisfy all equations by using the open parameters (e.g. α and the magnitude of $q(t)$ on each segment). If you can satisfy all optimality conditions, you have found a solution (a local minimum), though there may of course be other solutions (local minima). **(8 points)**

In addition to this, continue to work on your semester project.

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!