MATH 652: Optimization II

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Homework assignment 8 – due Thursday 4/8/2010

Problem 1 (An optimal control problem). If you put an amount $q$ of fertilizers in a flower pot at time $t = 0$, watering the plant will over time reduce the available amount to $a(t) = qe^{-t}$. A plant that starts at height $h(0) = 1$ at time $t = 0$ will grow according to

$$
\dot{h}(t) = a(t).
$$

If we want the flower to have height $h(4) = 4$ at time $t = 4$ (for example because we want to sell it, and this height is what the customer wants), then we need to ask how much fertilizer $q$ we need to put into the point at $t = 0$. This leads to the optimal control problem

$$
\min_{h(t), q \in \mathbb{R}} \frac{1}{2} (h(4) - 4)^2 = \frac{1}{2} \int_0^T (h(t) - 4)^2 \delta(t - 4) \, dt \quad \dot{h}(t) = qe^{-t},
$$

$$
h(0) = 1.
$$

Derive the optimality conditions for this problem and find a solution to it.

(4 points)

Problem 2 (An optimal control problem). The pilot of a plane wants to climb along a linear trajectory $\hat{z}(t) = 4t$ but there are updrafts and downdrafts that result in vertical forces $\cos(t)$. We will need to produce forces $q(t)$ so that the plane climbs along the desired route. Let’s model this problem as follows:

$$
\min_{z(t), q(t)} \int_0^T (z(t) - \hat{z}(t))^2 \, dt \quad \ddot{z}(t) = \cos(t) + q(t),
$$

$$
z(0) = 0, \quad \dot{z}(0) = 4.
$$

Reformulate the problem so that you have only first-order time derivatives by introducing a second variable $v(t)$ as in class. Formulate optimality conditions and find a solution to the optimization problem based on these.

(6 points)
**Problem 3 (More on this).** Describe what would have happened if the second initial condition had been

$$\dot{z}(0) = 0.$$ 

What would the optimality conditions have been in that case, and what does that mean for the solution.  

(2 points)

**Problem 4 (Even more on this).** Can you come up with a way how we could have worked with the originally posed formulation of Problem 2, i.e. without introducing a velocity variable $v(t)$? Hint: You need a variational form of the differential equation that contains two time derivatives, along with their initial conditions.  

(2 points)

*If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!*