

MATH 652: Optimization II

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Homework assignment 5 – due Tuesday 3/2/2010

Problem 1 (Duality 1). Consider the following problem:

$$\begin{aligned} \min_{x \in \mathbb{R}^2} x_1^2 + x_2 \\ x_2 \geq 1 \end{aligned}$$

Answer the following questions: (i) What is the solution of this problem? (ii) Formulate the dual problem. (iii) What is the solution of the dual problem? (iv) What is the value of the duality gap? **(4 points)**

Problem 2 (Duality 2). Consider the following problem:

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \sum_{i=1}^n \cosh x_i \\ (x_i - 1)^4 \leq 2 \quad \forall i = 1, \dots, n. \end{aligned}$$

Formulate the dual problem. Prove that this problem has a duality gap of zero. **(4 points)**

Problem 3 (Dual problem of a linear problem). Consider the following linear problem ($c \in \mathbb{R}^n$, $A_1 \in \mathbb{R}^{m_1 \times n}$, $A_2 \in \mathbb{R}^{m_2 \times n}$, $A_3 \in \mathbb{R}^{m_3 \times n}$, $b_i \in \mathbb{R}^{m_i}$):

$$\begin{aligned} \min_{x \in \mathbb{R}^n} c^T x \\ A_1 x = b_1 \qquad A_2 x \geq b_2 \\ A_3 x \leq b_3 \qquad x \geq 0. \end{aligned}$$

Formulate the dual of this problem. **(4 points)**

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!