Problem 1 ($l_\infty$ minimization). Assume you are given the following time series:

<table>
<thead>
<tr>
<th>$t_i$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_i$</td>
<td>1.1</td>
<td>1.9</td>
<td>2.8</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Consider the problem of fitting a line $y(t) = at + b$ through this data set. One way to do so is to ask for that set of parameters $x = \{a, b\}$ for which the maximal deviation $f(x) = \max_{i=1\ldots4} |y_i - y(t_i)|$ is minimal. Note that the right hand side depends on $x$ through the equation for $y(t)$.

Re-state this problem as a linear programming problem by introducing a (single) slack variable $s$. This way, you get a linear optimization problem in three variables: $a, b, s$.

While we will leave finding the solution of such problems for later, try to visualize the feasible set of this problem, i.e. the set of all points $\{a, b, s\}$ that satisfy the constraints of the re-formulated problem. (6 points)

Problem 2 (A network problem). Consider the following network problem (node numbers are given in boxes):

We want to consider the problem of finding the maximal data rate for sending data from node 1 to node 4. Bandwidths of all connections are shown along edges of the graph. Edges not shown have a zero bandwidth.

Consider the formulation for the network capacity problem given on slides 14–16 in the lecture notes. Write finding the maximal data rate as a linear optimization problem

$$\min_x c^T x \quad \text{subject to} \quad Ax \geq b.$$ 

State explicitly what variables make up the vector $x$, and state the elements of the vectors $b, c$ and the matrix $A$. (Hint: $x$ will be a 6-dimensional vector.) Can you guess the solution of of this problem? (6 points)