

In class, we have derived that the probability for a single nucleus to be around at time t equals $p(t) = e^{-kt}$ for some value of k that depends on the material under consideration. We then went on to compute the probability $P_n(t)$ to find exactly n nuclei if there were N to begin with. Let us visualize these probabilities for $N = 5$, $k = 1$:

$$P := (n, t) \rightarrow \frac{N!}{n! (N-n)!} \cdot p(t)^n \cdot (1-p(t))^{N-n};$$

$$(n, t) \rightarrow \frac{N! p(t)^n (1-p(t))^{N-n}}{n! (N-n)!} \tag{1}$$

$$p := t \rightarrow \exp(-k \cdot t);$$

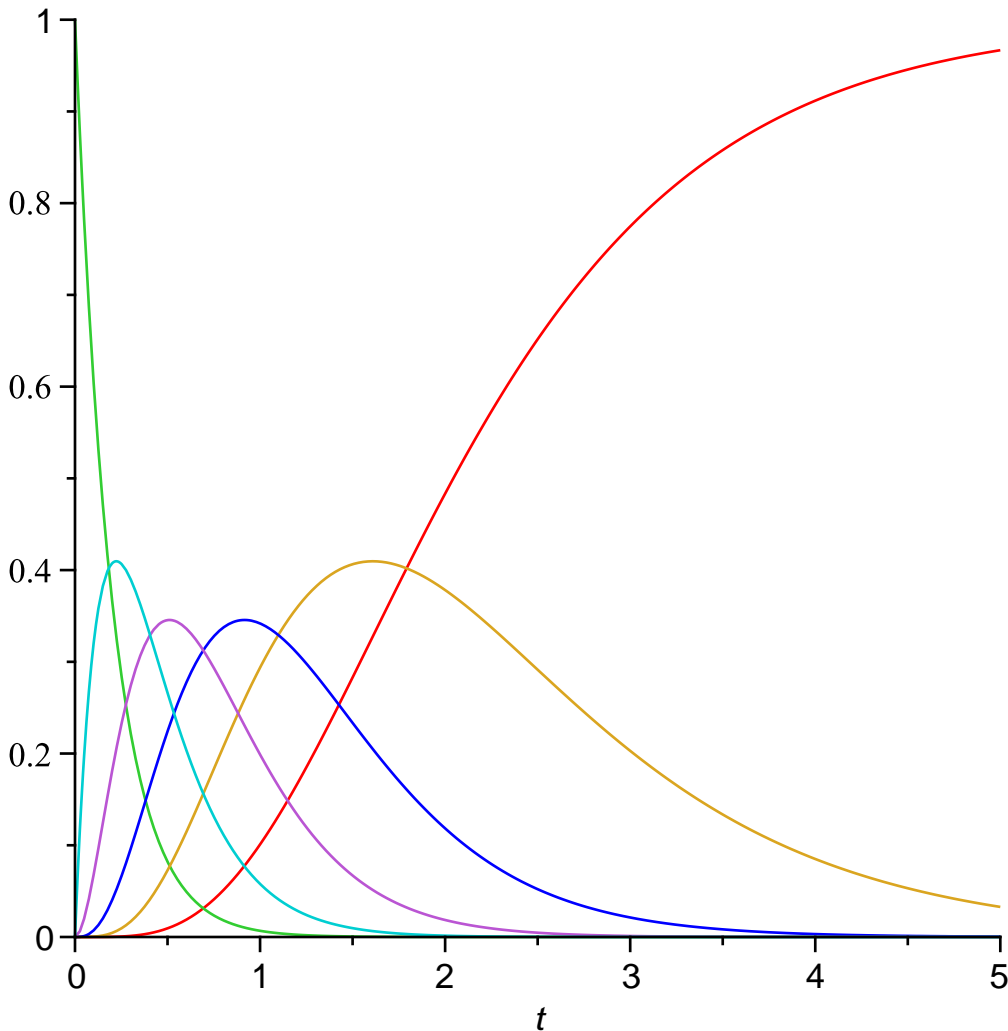
$$t \rightarrow e^{-kt} \tag{2}$$

$$N := 5; k := 1;$$

$$5$$

$$1 \tag{3}$$

`plot({seq(P(i, t), i=0..N) }, t=0..5);`



As you can see, at the beginning we know for sure (with probability one) that there are 5 nuclei; this probability drops rapidly and the probability to find 4, 3, 2 or less nuclei first grows and then decreases

again as it becomes more and more likely that in fact no nuclei are around any more (i.e. $P_0(t)$, the red curve, grows).