MATH 442: Mathematical Modeling

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Homework assignment 4 – due 9/30/2010

Problem 1 (Predator prey models). The Lotka Volterra model we discussed in class has the form

$$m'(t) = (a - \hat{\gamma} w(t))m(t) \qquad m(0) = m_0,$$
$$w'(t) = (\hat{\delta} m(t) - \epsilon)w(t) \qquad w(0) = w_0.$$

Let's think of $m(t), w(t)$ as the populations of moose and wolves, respectively. Reasonable values for the parameters above may be:

- $a = 0.4$. We derived a as $a = \alpha - \beta$ where α is the birth rate and β the rate of natural deaths (not related to wolves). Let's say for simplicity each two moose may have 1 offspring per year, then $\alpha = 0.5$. If left alone, they may live for 10 years, making $\beta = 0.1$.
- $\hat{\gamma} = 0.002$. We had that $\hat{\gamma} w(t)$ is the death rate of moose if there are $w(t)$ wolves around. Let's say, for example, that if there are 100 wolves around then moose will live on average for five years, i.e. $\hat{\gamma} 100 = \frac{1}{5}$, yielding the value above.
- $m_0 = 1000$ is the initial number of moose.
- $\hat{\delta} = 0.0025$. We had that $\hat{\delta} m(t)$ is the birth rate of wolves. Let's assume that they can have 5 births per year per couple of wolves if there are 1000 moose around to eat.
- $\epsilon = 0.1$. This is the natural death rate of wolves and we get this value by assuming that they live for 10 years.
- $w_0 = 50$ is the initial number of wolves.

Maple will not be able to find an analytic solution to these equations, but you can instruct the dsolve function to find a numeric solution, i.e. an object that if you give it the time it will return to you the number of moose or wolves.

Use the result of this numerical computation to plot the wolf and moose populations over 1000 years. Show your results in two ways: as a graph that shows both $m(t)$ and $w(t)$ as a function of time in the same plot; and as a phase plot as discussed in class. Describe in words what you observe. To what extent does this make sense?

Note: Getting maple to plot the results of odesolve is tricky, but possible. You may wish to consider a combination of the `output=listprocedure` flag to dsolve, and the rhs() function. An alternative is the odeplot function. Read through maple's help pages to see how to use these. **(6 points)**

Compare these results to what you get if you choose the natural lifespan of moose (without wolves) to be 2.5 years and 100 years, respectively. **(2 points)**

In each of the three cases you investigated, what is the minimum number of moose you observe over the course of the 1000 years? Does this make sense? **(2 points)**

Problem 2 (Competition and symbiosis models). In class we talked about the construction of a model that describes the results of the interaction of a predator and its prey on the population numbers of the two. But there are other interactions between species that do not involve one eating the other. For example, different species might compete for the same resource (e.g. competition for light or nutrients between plants), or they may even be beneficial to each other (read up on the articles on "symbiosis" and "lichen" on wikipedia).

Part a: In analogy to the Lotka Volterra predator prey model, develop a model that describes the competition between two species for the same resource. Your model should have the characteristics of the logistic model, i.e. it should have a separate carrying capacity for each species. However, the carrying capacity for each species should depend on the fraction of the resource this species can get. For each coefficient that appears in your model, discuss what its sign should be and why. **(4 points)**
Describe a set of coefficients and initial conditions that could make sense for a competitive system of your choice and use maple to solve it numerically for a sufficiently long time to see the dynamics of the system. Plot the populations of the two species over this time interval. **(3 points)**

Part b: In analogy to the Lotka Volterra predator prey model, develop a model that describes the symbiosis of two species. Your model should contain a carrying capacity of the ecosystem for each of the two partners in the symbiontic system (they do not compete for the same resources but the resource each needs is limited anyway). Your model should also account for the fact that each of the two species needs the other and that they survive best if their population numbers stand in a certain ratio. For each coefficient that appears in your model, discuss what its sign should be and why. **(4 points)**
Describe a set of coefficients and initial conditions that could make sense for a

symbiontic system of your choice and use maple to solve it numerically for a sufficiently long time to see the dynamics of the system. Plot the populations of the two species over this time interval. **(3 points)**

I try to be as good a teacher as possible, but to succeed in this goal I need feedback from those who see me teach, i.e. you. If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, group work vs. whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!