MATH 412: Theory of Partial Differential Equations

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Homework assignment 7 – due Thursday 10/18/2007

Problem 1 (Linearity of the Fourier transform). Solve Problem 3.2.3 in the book. **(2 points)**

Problem 2 (Fourier series). Derive the Fourier series on $[-\pi, \pi]$ of the function $f(x) = x$. From this series, derive the Fourier series of $F(x) = x^2/2$ without using the formulas $\frac{1}{L}\int_{-L}^{L} F(x)\cos nx\, dx$ (and similar for the sine terms) to compute the coefficients A_0, A_n, B_n of the second series. **(4 points)**

Problem 3 (Fourier series). Derive the Fourier series on $[-\pi, \pi]$ of the function $F(x) = x$ (it is the same as in Problem 2). State whether you can derive the Fourier series of the function $f(x) = 1$ from it by differentiating each term in the series of $F(x)$ individually. State the Fourier series of $f(x)$. **(3 points)**

Problem 4 (Fourier series). Generate a computer plot of the partial sums $A_0 + \sum_{n=1}^{10} A_n \cos(nx) + B_n \sin(nx)$ consisting of the first 10 terms for the Fourier series of the functions $f(x) = 1, F(x) = x$, where the Fourier series is calculated over the interval $-\pi \dots \pi$. Plot these Fourier series on the larger interval $-2\pi \dots 2\pi$. Also generate a plot of the partial sum $\sum_{n=1}^{10} -A_n n \sin(nx) + B_n n \cos(nx)$ with the coefficients A_n, B_n of the Fourier series of $F(x) = x$ (this is the term-by-term differentiated Fourier series of $F(x)$.) **(3 points)**

Problem 5 (Fourier series). Calculate or look up the Fourier series on the interval $-\pi \dots \pi$ of the function

$$f(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x \leq 0 \end{cases}$$

Generate plots of the partial sums $S_N(x) = A_0 + \sum_{n=1}^{N} A_n \cos(nx) + B_n \sin(nx)$ consisting of the first N terms, for each value $N = 2, 5, 10, 20, 50$. Try to determine the maximal difference $|f(x) - S_N(x)|$ numerically or graphically. We know that for $N \to \infty$, $S_N(x) \to f(x)$ almost everywhere; is this consistent with your results? **(4 points)**