MATH 417: Numerical Analysis

Instructors: Prof. Wolfgang Bangerth, Prof. Guido Kanschat
bangerth@math.tamu.edu, kanschat@math.tamu.edu

Teaching Assistants: Seungil Kim, Yan Li
sgkim@math.tamu.edu, yli@math.tamu.edu

Homework assignment 8 – due 11/2/06 and 11/6/06

Problem 1 (Condition numbers). Calculate the condition numbers \( \kappa(A) = \|A\| \|A^{-1}\| \) with respect to the \( l_1, l_\infty \) and \( l_2 \) norms for the matrix

\[
A = \begin{pmatrix}
1 & 0.001 \\
0.999 & 1
\end{pmatrix}.
\]

(5 points)

Problem 2 (Error propagation). With the matrix from Problem 1, consider the solutions \( x, \tilde{x} \) of the following linear systems:

\[
Ax = b, \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix},
\]

\[
A\tilde{x} = \tilde{b}, \quad \tilde{b} = \begin{pmatrix} 1 \\ 1.001 \end{pmatrix}.
\]

(Imagine the former to be the exact right hand side, and the latter to be one that is contaminated by measurement uncertainty, statistical error, etc.)

Solve for \( x \) and \( \tilde{x} \). Calculate the relative difference in the right hand side \( \epsilon_r = \|b - \tilde{b}\|/\|b\| \) and the relative error \( e_r = \|x - \tilde{x}\|/\|x\| \) in the solution, each for both the \( l_2 \) and the \( l_\infty \) norm.

Using your result from Problem 1, do \( \epsilon_r \) and \( e_r \) satisfy the estimates discussed in class? (5 points)

Problem 3 (Lagrange interpolation).

(a) Compute the Lagrange interpolation polynomials \( L_{4,k}, k = 0 \ldots 3 \), for the points \( x_0 = 1, x_1 = 2, x_2 = 1.5 \) and \( x_3 = 1.6 \).

(b) Calculate the interpolating polynomial for the data set where \( y_k = \log x_k \) at the four points \( x_k \). Write the polynomial in the form \( a_3x^3 + a_2x^2 + a_1x + a_0 \).

(c) The polynomial calculated in (b) by construction interpolates the function \( f(x) = \log x \). Compute the maximal error on the interval \([1, 2]\). (6 points)