MATH 417: Numerical Analysis

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Homework assignment 4 – due 9/28/06 and 10/2/06

Problem 1 (Secant method). This problem is an example of finding the root of a function f that is only given in form of a procedure, a likely case in applications, instead of as a closed form expression.

In order to define the function $g(x)$, consider the following iteration: set $a_0 = 1$ and compute the values a_i by the following iteration:

$$a_i = a_{i-1} + \frac{x \sin a_{i-1} + x}{10}.$$

Clearly, we can compute a_1 from a_0 for each value of x. Similarly, we can compute a_2 from a_1, and so on. Now, let $g(x)$ be the function whose value equals a_{10} for any given value of x.

a) Write a program function that given a value x returns $g(x) = a_{10}$ by computing the iteration above.

b) Assume we want to solve the equation $f(x) = 0$ where $f(x) = g(x) - 3$. State why Newton's method may be ill-suited for this task.

c) Write a program that finds a root of $f(x) = g(x) - 3$ up to 6 digits accuracy using the secant method. **(6 points)**

Problem 2 (Root finding methods). Compare, in words, the bisection method, Newton's method, and the secant method with respect to the following criteria: reliability of finding a root of a function, speed of convergence, complexity (i.e., a method is better if it needs fewer evaluations of $f(x)$ per iteration, or if it only needs function values rather than derivatives).

(3 points)

(please turn over)

Problem 3 (Gaussian elimination). Solve (on paper, showing the individual steps) the following system of linear equations using Gaussian elimination:

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

Verify that your result is correct.

(The matrix is the example is the so-called Hilbert matrix, with entries $H_{ij} = \frac{1}{i+j-1}$. It has a number of nasty properties that make it a good testcase for matrix algorithms.) **(3 points)**

Problem 4 (Gaussian elimination). Using Gaussian elimination, it is simple to solve the following problem

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

One would eliminate the occurrence of x_1 in the second equation by subtracting the first from the second equation, arriving at a diagonal matrix.

Describe what happens if the system instead looked like this:

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Does the algorithm still work? If not, propose a remedy. **(2 points)**