MATH 412: Theory of Partial Differential Equations

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Homework assignment 6 – due Thursday 10/12/2006

Problem 1 (Fourier series). The Fourier series on $[-L, L]$ of a function $f(x)$ that is piecewise smooth is given by

$$A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L}$$

where

$$A_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx,$$

$$A_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx, \qquad B_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx.$$

Calculate the Fourier series on $[-\pi, \pi]$ of the function

$$f(x) = \begin{cases} 1 & \text{for } x \geq 0, \\ -1 & \text{for } x < 0. \end{cases}$$

(5 points)

Problem 2 (Gibbs phenomenon). Using a computer graphing program such as Maple, Matlab, or Mathematica (or whatever else you deem fit for the task), graph for the range $x \in [-\pi, \pi]$ the following:

- $f(x)$ and the first 3 terms of its Fourier series;
- $f(x)$ and the first 6 terms of its Fourier series;
- $f(x)$ and the first 15 terms of its Fourier series;
- $f(x)$ and the first 30 terms of its Fourier series.

(Give us a printout of the plot or plots.) You will see that the plots of the first terms of the Fourier series approximate $f(x)$ increasingly well, but that there are over- and undershoots around the location where $f(x)$ has a jump (i.e. at $x = 0$). These oscillations are called *Gibbs phenomenon*.

Conjecture what the Fourier series converges to for points $x < 0$, $x = 0$, and $x > 0$. **(5 points)**

(please turn over)

Problem 3 (Periodic continuation). Repeat the plots you already generated for Problem 2, but this time show $f(x)$ and the first 3, 6, 15, and 30 terms of its Fourier series in the interval $[-3\pi, 3\pi]$. Interpret what you see.

(3 points)

Problem 4 (Linearity of the Fourier transform). Solve Problem 3.2.3 in the book. **(2 points)**