MATH 412: Theory of Partial Differential Equations

Lecturer: Prof. Wolfgang Bangerth
Blocker Bldg., Room 507D
(979) 845 6393
bangerth@math.tamu.edu
http://www.math.tamu.edu/~bangerth

Homework assignment 2 – due Thursday 9/14/2006

Problem 1 (Linear operators). Solve exercise 2.2.2 in the book.
(2 points)

Problem 2 (Linear operators). Solve exercise 2.2.5 in the book.
(3 points)

Problem 3 (Sign of eigenvalues). Consider the equation

$$\frac{d^2\phi(x)}{dx^2} + \lambda\phi(x) = 0.$$

Determine whether $\lambda > 0, \lambda \geq 0, \lambda \leq 0$, or $\lambda < 0$ based on the technique shown in class, for each of the following sets of boundary conditions:

$$\begin{aligned} a) \qquad & \phi(0) = \phi(\pi) = 0, \\ b) \qquad & \frac{\partial\phi}{\partial x}(0) = \frac{\partial\phi}{\partial x}(1) = 0. \end{aligned}$$

(3 points)

Problem 4 (Solutions of the heat equation). Solve exercise 2.3.3 (a) and (b) in the book. Note that similar problems are solved in the main text.
(2 points)

Problem 5 (Orthogonality of trigonometric functions). Solve exercise 2.3.5 in the book.
(2 points)

(please turn over)

Problem 6 (Some basics). Let $u(x,t)$ be a function of space and time. Prove the following identities by going back to the basics of multivariate calculus (definition of derivatives, definition of integrals, etc):

a) $$\frac{d}{dt}\int_0^L u(x,t)\,dx = \int_0^L \frac{\partial u(x,t)}{\partial t}\,dx,$$

b) $$\frac{d}{dx}\int_0^L u(x,t)\,dx = 0,$$

c) $u(0,t) = u(L,t)$ implies $$\int_0^T\int_0^L \frac{\partial u(x,t)}{\partial x}\,dx\,dt = 0,$$

d) $u(x,t) = x^2t^2$ implies $$\int_0^T\int_0^L \frac{\partial u(x,t)}{\partial x}\,dx\,dt = \frac{L^2T^3}{3},$$

e) $$\int_0^L u(x,T)\,dx = \int_0^L u(x,0)\,dx + \int_0^T\int_0^L \frac{\partial u(x,t)}{\partial t}\,dx\,dt.$$

(5 points)