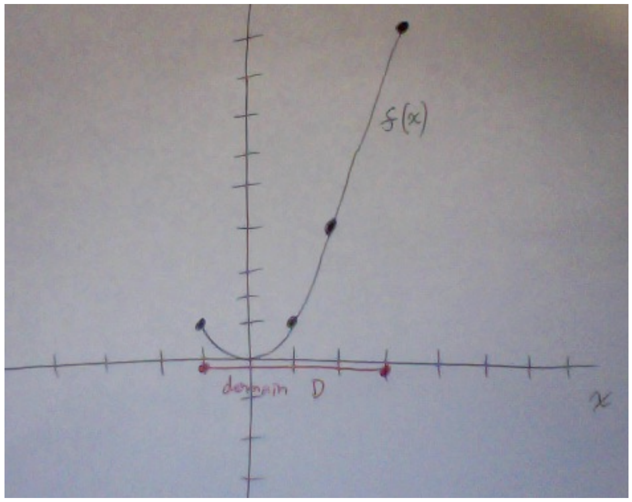
Can you have a local max at an endpoint?

Consider the function $f(x) = x^2$ defined on the closed interval $-1 \le x \le 3$. Is $c = -1$ a local max? It depends on who you ask. The book says no, and I say yes!



What's the definition of a local max? Let $f(x)$ be a function defined on a domain D, and let c be in D. We say that f has a *local max* at c if $f(c) \ge f(x)$ for all x near c. However, the book and I interpret this definition differently!

The book's interpretation of this definition is: we say that f has a *local max* at c if $f(c) \ge f(x)$ for all x near c, even for those x not in D.

My interpretation of this definition is: we say that f has a *local max* at c if $f(c) \ge f(x)$ for all x in D near c.

In our example above, we have $f(x) = x^2$, domain D is $-1 \le x \le 3$, and $c = -1$. The book says that since -1.01 is not in the domain D of f, $f(-1.01)$ is not defined, and hence the inequality $f(-1) \ge f(-1.01)$ makes no sense and cannot be satisfied. Therefore $c = -1$ is not a local max by the book's interpretation. Conversely, I say that $c = -1$ is a local max because for all x near -1 in the domain D, we have $f(-1) \ge f(x)$.

In summary, the book says that neither $c = -1$ nor $c = 3$ are local maxima, but that $c = 3$ is an absolute max. Conversely, I say that -1 and 3 are both local maxima and that $c = 3$ is also an absolute max. With the book's interpretation, not every absolute max is a local max! But with my interpretation, every absolute max is also a local max. This is one reason why I prefer my interpretation.

In most problems, this distinction doesn't matter. Whether you call an endpoint a local max or not doesn't affect how you optimize a function on an interval. However, this distinction does matter on homework problems §4.2 #6, and §4.3 #12. Use whichever interpretation you prefer, and I'll tell the TAs to accept both answers (i.e. either saying $c = -1$ in the example above is a local max or not). If you want to make your homework solutions super clear, feel free to write something like "I'm using the book's convention where endpoints can't be local maxima" or "I'm using Henry's convention where endpoints can be local maxima" next to your answers for §4.2 #6 and §4.3 #12.

Let me know if you have any questions on this.