CSU Math 571 **Practice Midterm**

Name: ________________________________

- No notes, books, calculators, or other electronic devices are permitted.
- For the True/False question #4, just say "True" or "False". No justification is required, and no partial credit is available.
- Unless stated otherwise, points are distributed evenly between multiple parts of a problem.
- Please sign below to indicate you accept the following statement:

  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

$\boxed{1}$ (a) (6 points) Let X be the union of path-connected open sets A_α each containing the basepoint $x_0 \in X$. Suppose each two-fold intersection $A_\alpha \cap A_\beta$ is path-connected, and furthermore each three-fold intersection $A_\alpha \cap A_\beta \cap A_\gamma$ is path-connected. Then van Kampen's Theorem provides a group that $\pi_1(X)$ is isomorphic to. What is this group?

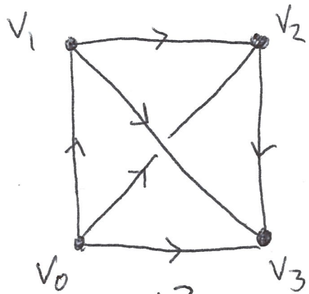
(b) (4 points) Let $X = \vee_\alpha X_\alpha$ be a wedge sum of path-connected CW complexes. Prove that $\pi_1(X) \cong *_\alpha \pi_1(X_\alpha)$.

$\boxed{2}$ *Remark: For this problem you do not need to prove your answers are correct.*
Let $p\colon S^1 \times \mathbb{R} \to S^1 \times S^1$ be the covering space map defined by $p(e^{2\pi i \cdot s}, t) = (e^{2\pi i \cdot 3s}, e^{2\pi i \cdot t})$ for $e^{2\pi i \cdot s} \in S^1$ and $t \in \mathbb{R}$.

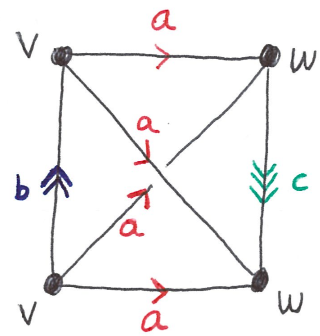
(a) What group is $\pi_1(S^1 \times S^1)$ isomorphic to? Identify the subgroup of $\pi_1(S^1 \times S^1)$ that is isomorphic to $p_*(\pi_1(S^1 \times \mathbb{R}))$.

(b) Is this covering space normal? Identify the group of deck transformations.

$\boxed{3}$ Let X be the Δ-complex obtained from the 3-simplex Δ^3 by identifying $[v_0, v_2, v_3]$ with $[v_1, v_2, v_3]$, and also $[v_0, v_1, v_2]$ with $[v_0, v_1, v_3]$, where both of these identifications are done in an orientation-preserving way.



Δ^3



X

After these identifications, we have

- two 0-simplices v and w,
- three 1-simplices a, b, c,
- two 2-simplices L (the identification of $[v_0, v_2, v_3]$ with $[v_1, v_2, v_3]$) and R (the identification of $[v_0, v_1, v_2]$ with $[v_0, v_1, v_3]$), and
- a single 3-simplex (call it T).

Compute the simplicial homology $H_i(X)$ for $i = 0, 1, 2, 3$.

Remark: I advise you to do the algebra as opposed to trying to visualize the space (which is not easy).

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$\boxed{4}$ Just say "True" or "False". No justification is required; no partial credit is available.

(a) Let $\tilde{X}$ be a torus. Then there exists a space X and a covering space map $p\colon \tilde{X} \to X$ with $p_*(\pi_1(\tilde{X})) \cong \mathbb{Z}$.

(b) There is a covering space map $p\colon M_{22} \to M_2$, where M_g is the closed orientable surface of genus g (i.e., the "g-holed torus").

(c) There exists a Δ-complex X and a non-negative integer n such that the cardinality of the simplicial chain group $\Delta_n(X)$ is larger than the cardinality of the singular chain group $C_n(X)$.

(d) Let X and Y be topological spaces with Y path-connected and nonempty, and let $X \coprod Y$ be their disjoint union. Then there is a group isomorphism

$$\tilde{H}_0(X \coprod Y) \cong H_0(X)$$

between the reduced 0-dimensional homology of $X \coprod Y$ and the 0-dimensional homology of X.

(e) Let G be an arbitrary group. Then G can be obtained as the fundamental group of some CW complex.

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