CSU Math 571
Spring 2018

Homework 2

Due Friday, January 26 at the beginning of class

Reading. Section 1.1 of Hatcher, which is material you've seen before, focusing on the proof of Theorem 1.7 (the fundamental group of the circle).

Problems.

1. Give an example of a connected space X, a subspace $A \subseteq X$, and a map $r\colon X \to A$ that is a retract but which does not come from a deformation retraction. Prove your answer is correct.

   Remark: These terms are all defined on pages 2-3 of our book, which I encourage you to read.

   Remark: In 571 you can of course use any machinery from 570.

2. The *Euler characteristic* of a finite CW complex X is $\chi(X) = \sum_i (-1)^i c_i$, where c_i is the number of i-cells of X. Alternatively see also the definition on page 6 of our book.

   (a) If X is S^n with a CW structure of a single 0-cell e^0 and a single n-cell e^n, then what is $\chi(X)$?

   (b) If X is S^n with a CW structure of two 0-cells, two 1-cells, $\ldots$, two n-cells, then what is $\chi(X)$?

   (c) Explain why Theorem 2.44 on page 146 of our book, which we haven't covered yet, says that you should have expected to get the same answer in (a) and (b).

   (d) Any simplicial complex has a CW complex structure with one i-cell for each i-simplex. Find the Euler characteristic of the n-simplex Δ^n by counting c_i for each i, and then computing the alternating sum $\chi(\Delta^n) = \sum_i (-1)^i c_i$. How in the world do you simplify that long alternating sum?

   (e) Instead, now find the Euler characteristic of the n-simplex by remarking that Δ^n is homotopy equivalent to a simpler space (no proof needed), and then computing the Euler characteristic of that simpler space.

3. Show that S^∞ is contractible. I encourage you to consult Example 1B.3 on page 88 in our book!

   Remark: In Example 1B.3 in our book, $\mathbb{R}^\infty = \cup_n \mathbb{R}^n$ *is the set of all sequences* $x = (x_1, x_2, x_3, \ldots)$ *where all but finitely many of the coordinates are zero. The sphere* S^∞ *is the subset of such sequences for which* $\|x\| = \sqrt{\sum_{i=1}^\infty x_i^2} = 1$.

   Remark: So long as you write down maps that are continuous, in 571 you essentially never need to prove that they're continuous.