CSU Math 570
Fall 2017

Homework 3

Due Friday, September 8 at the beginning of class

Reading.
Attend Tour de Fat (honor system). Finish Chapter 3 through Page 77 (for now we're skipping the section on Topological Groups and Group Actions), Read Chapter 4 through page 93.

Problems.

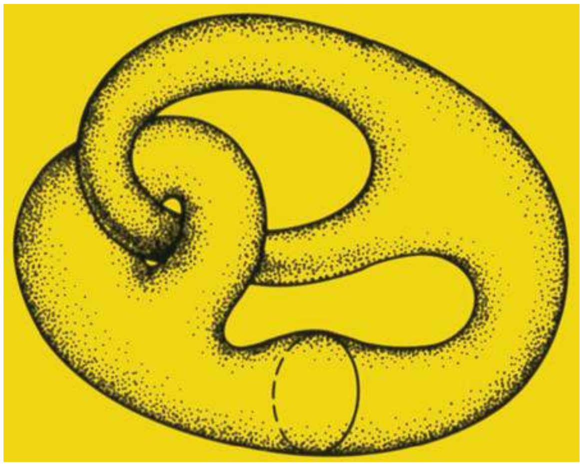
1. Let X be a topological space and let $A \subseteq X$ be a subset.
   - The *interior of* A *in* X, denoted by $\text{Int}A$, is the largest open set in X contained inside of A:
   $$\text{Int}A = \cup\{U \subseteq X \mid U \subseteq A \text{ and } U \text{ is open in } X\}.$$
   - The *boundary of* A *in* X is $\partial A = \overline{A} \cap \overline{X \setminus A}$ (this is equivalent to the slightly different definition on page 24 of our book).

   (a) Prove that a point $x \in X$ is in ∂A if and only if every neighborhood of x contains both a point of A and a point of $X \setminus A$.

   (b) Prove that A is open in X if and only if A contains none of its boundary points, i.e. $A \cap \partial A = \emptyset$.

2. Let X be a topological space and let $S \subseteq X$. Prove that the subspace topology on S is indeed a topology (i.e. satisfies the definition on page 20 of our book).

3. Read and understand Proposition 2.44 and its proof in our book, which describes when a collection $\mathcal{B}$ of subsets of a set X is a valid basis for some topology on X. Now, suppose that $X_1, \ldots, X_n$ are topological spaces. Use Proposition 2.44 to prove that the basis $\mathcal{B} = \{U_1 \times \ldots \times U_n \mid U_i \text{ is open in } X_i \text{ for all } i\}$ is indeed a valid basis for some topology on $X_1 \times \ldots \times X_n$ (called the *product topology*).



4. The above images are of a 2D surface which is a 2-holed torus. On the left the holes appear to be linked, but on the right they do not. However, there is a way to bend and stretch this shape in $\mathbb{R}^3$ to get from the surface on the left to the one on the right (imagine the shape is made of a very flexible rubber or play-doh which you are allowed to bend or stretch but not tear). Draw a deformation (a sequence of pictures) showing how to do this.

   (This is called an *ambient isotopy in* $\mathbb{R}^3$ between the two shapes. We won't discuss ambient isotopies much in this class.)