CSU Math 472
Fall 2016

Homework 6

Due Friday, October 7 at the beginning of class

Reading.
Sections 3.5, 3.6

Problems.

1. Let X be the set of real numbers equipped with the finite-complement topology (a set is open if its complement is finite or all of X). Is X compact? Prove it.

   Remark: Don't approach this problem by asking whether or not X is closed and bounded, since X cannot be thought of as a subspace of Euclidean space.

2. Each of the following spaces X can be thought of as a subpace of Euclidean space $\mathbb{R}^n$ for some n. For each space, say either

   - "X is compact."
   - "X is not compact because it is not closed."
   - "X is not compact because it is not bounded."
   - "X is not compact because it is neither closed nor bounded."

   You do not need to justify your answer.

   (a) $X = \mathbb{Q}$ is the space of rational numbers.
   (b) X is the n-sphere S^n with a finite number of points removed.
   (c) X is the torus $S^1 \times S^1$ with an open disk removed.
   (d) X is the Klein bottle.
   (e) X is the Möbius band with its boundary circle removed.

3. Let X be a topological space. The *diagonal* map $\Delta\colon X \to X \times X$ is defined by $\Delta(x) = (x, x)$. Prove that Δ is continuous.

   Revised Hint: Note that if $f\colon Z \to Y$, and if β is a base for the topology on Y, then f is continuous if the preimage $f^{-1}(B)$ of every set $B \in \beta$ is open in Z. Why is this? An arbitrary open set U in Y can be written as a union of sets in β; hence $f^{-1}(U)$ is a union of open sets in X of the form $f^{-1}(B)$ with $B \in \beta$; hence $f^{-1}(U)$ is open in Z.

   Back to the homework problem above: Start with a basic open set $U \times V$ in $X \times X$. It follows that $\Delta^{-1}(U \times V) = U \cap V$. Since U and V are each open in X, it follows *that $U \cap V$ is open in X. Hence we are done — Δ is continuous since the preimage of every basic open set is open.*

4. Choose any old homework or exam problem. State both the problem and the homework/exam number. Write out a solution that is as clear as possible, with no extraneous steps.

   Remark: You are encouraged to look at and mimic my solutions. However, do not look at my solution simultaneously while you are writing yours. Alternate between looking and writing.