CSU Math 472
Fall 2016

Homework 3

Due Friday, September 9 at the beginning of class

Reading.
Sections 2.1, 2.2

Problems.

1. Stereographic projection $\pi\colon S^2 \setminus \{(0,0,1)\} \to \mathbb{R}^2$ maps the sphere minus the north pole to the plane; see Figure 1.24 in our book. There are various ways to define this map π, using a variety of coordinate systems (`https://en.wikipedia.org/wiki/Stereographic_projection#Definition`).
   (a) Derive one such formula for π of your choosing. You can be terse here and not explain everything.
   (b) Check that π is a homeomorphism.
2. Show that Figure 1.9(d) and Figure 1.9(c) are homeomorphic, as follows.
   (a) Deduce from #1 that sterographic projection induces a homeomorphism $\pi\colon \mathbb{R}^2 \setminus \{(0,0,\pm 1)\} \to \mathbb{R}^2 \setminus \{(0,0)\}$ from the sphere minus the north and south poles to the plane with the origin removed.
   (b) Show that $\mathbb{R}^2 \setminus \{(0,0)\}$ and $\{(x,y) \mid 1 < \sqrt{x^2+y^2} < 3\} = \{re^{i\theta} \mid 1 < r < 3\}$ are homeomorphic.
3. Define $f\colon [0,1) \to S^1 = \{(x,y) \mid x^2+y^2 = 1\}$ by $f(x) = e^{2\pi i x} = (\cos(2\pi x), \sin(2\pi x))$. Function f is bijective and continuous; you do not need to prove this. Find a point $x \in [0,1)$ and a neighborhood N of x in $[0,1)$ such that $f(N)$ is not a neighborhood of $f(x)$ in S^1. Deduce that f is not a homeomorphism.
4. Using the intuitive notion of connectedness (which we haven't yet defined), prove that a circle and a circle with a spike attached (Figure 1.26) cannot be homeomorphic.
5. Make a Möbius band out of a strip of paper, and then cut it along its central circle (don't turn this in).

   Now, draw pictures to show that identifying diametrically opposite points on one of the boundary circles of the cylinder creates the Möbius band. That is, if you take the cylinder $S^1 \times [0,1] = \{(x,y,z) \mid x^2+y^2 = 1 \text{ and } 0 \le z \le 1\}$ and identify each $(x,y,1)$ with $(-x,-y,1)$, then you get the Möbius band.