CSU Math 472
Fall 2016

Homework 2

Due Friday, September 2 at the beginning of class

Reading.
Sections 1.3, 1.4, 1.5, 1.6

Problems.

1. Use the definition of a homeomorphism to show that if X is homeomorphic to Y and Y is homeomorphic to Z, then X is homeomorphic to Z.

   For the time being, you may assume that X, Y, and Z are metric spaces, and use facts that you know about continuous functions between metric spaces. But this statement is also true if X, Y, and Z are more generally topological spaces.

2. Show that $(0, \infty)$ is homeomorphic to $(0, 1)$ as follows:

   (a) Define a map $f\colon (0, \infty) \to (0, 1)$ (or $f\colon (0, 1) \to (0, \infty)$ if you prefer). There are many possible choices for f.

   (b) Show that f is a bijection.

   (c) Show that f is continuous (using the definition of a continuous function from the reals to the reals, or the definition of a continuous function between metric spaces).

   (d) Show that f^{-1} is continuous.

3. Draw pictures illustrating how one can see Problem 11(c) on pages 22-23 of our book.

   I will draw such pictures in class for you on Monday. These pictures show not only that X is homeomorphic to Y, but also that X is *ambient isotopic* to Y.

4. (a) Let P be a regular polyhedron in which each face has p edges and for which q faces meet at each vertex. Using Euler's formula, prove that $\frac{1}{p} + \frac{1}{q} = \frac{1}{2} + \frac{1}{e}$.

   (b) Deduce that there are only three regular polyhedra that have triangular faces ($p = 3$).

   You may assume that these three regular polyhedra (the tetrahedron, octahedron, and icosahedron) exist, and that any regular polyhedron with the same combinatorics as the octahedron (for example) is indeed the octahedron.