

Name: \_\_\_\_\_

- For #1 and #2, write proofs that consist of complete sentences. For #3, #4, and #5, if you write the correct mathematical expression then you will get 100% credit even without any English words. But clear English explanations may help you get partial credit if you don't have the correct mathematical expression (and they may help you find the correct mathematical expression).
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:  
 "I will not give, receive, or use any unauthorized assistance."

Signature: \_\_\_\_\_

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 Prove that  $5n + 5 \leq n^2$  for all integers  $n \geq 6$ .

**CSU Math 301**

**Midterm 1**

- 2 More than 6,000 of the students attending a private university were born in one of the 50 US states. Prove that at least 121 of these students were born in the same state.

- 3 For this problem, assume the 365 days of the year are distinguishable (being assigned day 1 is different than being assigned day 2). How many ways are there to distribute the 365 days of the year to 5 distinguishable workers Alice, Bob, Carl, Diane, Eve, if each day must get distributed to exactly one person, and if ...
- (a) ... each worker gets exactly 73 days of work (not necessarily in a row)? Note that  $365 = 5 * 73$ . (3 points)
- (b) ... each worker gets assigned 73 days of work in a row, either days 1-73, 74-146, 147-219, 220-292, or 293-365 (3 points)?
- (c) ... the days can be assigned arbitrarily, and it is not required that a person gets assigned any days? For example, we could assign the first 300 days to Alice, and the last 65 days to Diane (4 points).

- 4 (a) The binomial theorem gives a formula for  $(x + y)^n$  using the binomial coefficients. State this formula.

- (b) Suppose  $A$ ,  $B$ , and  $C$  are sets such that  $|A| = 10$ ,  $|B| = 17$ ,  $|A \cap B| = 5$ , and  $|A \cup B \cup C| = 30$ . What are the possible values of  $|C|$ ?

- 5 (a) Baseball team A has 7 pitchers, and baseball team B has 5 catchers. The two teams have agreed to make a trade of the following form: 3 of team A's 7 pitchers will be traded for 2 of team B's 5 catchers. The local newspaper decides to make a ranked list of their top 4 favorite possible trades (3 of A's pitchers for 2 of B's catchers). How many different ranked lists could the newspaper make?
- (b) You want to buy a bouquet of 12 flowers. There are 4 different flower types to choose from: roses, tulips, violets, and daffodils. The bouquet with 4 roses and 8 tulips is the same as the bouquet with 8 tulips and 4 roses. How many different bouquets could you create?

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**Midterm 1**

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