

## Homework 5

Due Friday, September 23 at the beginning of class

**Reading.** Sections 3.5, 3.6, 4.1, 4.2

**Remark.** Make grammatically correct sentences by adding in just a few English words (for problems 1-4, but not for problem 5).

### Problems.

- (a) How many different anagrams (i.e. strings of length 10) can you form by rearranging the letters of the word HORSETOOTH? For example, THEHOSROOT and SOOOTHHER are two such rearrangements.
  - (b) How many ways are there to place 8 rooks on a chessboard with no two attacking each other if 4 are wooden, 2 are marble, and 2 are plastic? Rooks made from the same material are indistinguishable.
2. Prove by induction that  $2^n > 10n + 7$  for all  $n \geq 7$ .
3. You want to buy a bag of 20 berries, and there are 3 different kinds of berries to choose from. Any two berries of the same kind are indistinguishable. Buying 10 strawberries, 6 raspberries, and 4 blackberries gives the same bag as buying 6 raspberries, 10 strawberries, and 4 blackberries. How many different bags of 20 berries you buy?
4. You have a large supply of 5 different kinds of postcards (a ram, buffalo, fox, goat, and marmot postcard). You want to send one or two postcards to each of 8 different friends. How many different ways are there to do this?  
*Remark.* If Alice gets a ram postcard and Bob gets a buffalo postcard, that's different than if Alice gets a buffalo postcard Bob gets a ram postcard. Also, if Alice gets two postcards, then those postcards are required to be of two different kinds.
5. This last question is a "short answer" question, meaning no English words or explanations are required. Simply write down the correct mathematical expression.  
How many ways are there to distribute 13 pillows to 6 (distinguishable) people if ...
  - (a) ... the pillows are identical?
  - (b) ... the pillows are identical and every person must get at least one?
  - (c) ... the pillows are distinguishable and it is not required that every person get one? For example, we could give all 13 pillows to the first person.
  - (d) ... the pillows are distinguishable and we give 4 to the first person, 3 to the second person, 2 to the third person, 2 to the fourth person, 1 to the fifth person, and 1 to the sixth person?