

### Homework 3

Due Friday, September 9 at the beginning of class

#### Reading.

Sections 1.8, 2.1, 2.4

**Remark.** Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words.

#### Problems.

1. You have 16 dice, all of different colors. You want to loan a subset of dice to your friend, and the only requirement is that you must loan your friend an even number of dice. In how many different ways could you loan your friend an even number of dice?

*Remark.* Loaning your friend 0 dice, while mean, is still counted as loaning them an even number of dice.

2. Alice has six wristbands of different colors and Bob has seven necklaces of different colors. In how many ways can Alice trade two of her wristbands for three of Bob's necklaces?

3. (a) Suppose  $\binom{n}{k-1} = 120$  and  $\binom{n+1}{k} = 330$ . What is  $\binom{n}{k}$ ?

*Hint:* The best way to solve this problem does not require you to find  $n$  or  $k$ .

- (b) Give an algebraic proof that  $\binom{n}{2} + \binom{n+1}{2} = n^2$ .

*Remark.* I do *not* suggest using a proof by induction.

4. Seven friends are sharing three hotel rooms, where the first hotel room fits 1 person, the second fits 2 people, and the third fits the remaining 4 people. They are staying in the hotel for four nights in a row, and they are allowed to rearrange room assignments every night. In how many different ways can the seven friends create a four-night schedule of room assignments?

*Remark:* The first-night room assignment

Room 1: A, Room 2: B, C, Room 3: D, E, F, G

is the same as the first-night room assignment

Room 1: A, Room 2: C, B, Room 3: E, F, D, G.

Also, this first-night room assignment is allowed to be reused for later nights.

5. Let  $A$  and  $B$  be sets with  $|A \cup B| = 10$  and  $|B| = 6$ .

- (a) What are all the possible values of  $|A|$ ?
- (b) What are all the possible values of  $|A \cap B|$ ?
- (c) What are all the possible values of  $|A \setminus B|$ ?